LETTER

Accurate Source Model for MoM Analysis of Linear Antennas by Using Sinusoidal Reaction Matching Technique

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SUMMARY A new source model for MoM analysis by using sinusoidal reaction matching technique is proposed for linear antenna analysis. This source model assumes a constant feeding gap and uniform electric field distribution inside the gap. The analysis results are compared with the results of the conventional models and measurement. It is found that the new model can incorporate the effect of the length of driving gap and is more accurate and more stable than that from the conventional source models. The proposed source model is simple and easy to use. This source model, together with the full kernel formulation, makes it possible to analyze the linear dipole antennas with no limitation of ratio of segment length to radius.

key words: *method of moment, source model, feed, linear antenna*

1. Introduction

Since Richmond and Geary published the rigorous expressions for mutual impedance of nonplanar-skew sinusoidal monopoles [1], [2], the method of moment (MoM) by using sinusoidal reaction matching has been used extensively in analyzing the thin wire antennas because of its high accuracy and stability [3], [4].

Many types of source models have been applied to the MoM. The most commonly used source model is the so-called delta gap source model. According to the assumption of this source model, the element of the voltage vector corresponding to the driving segment has unit value and the other elements have values of 0 voltage. This model has been used extensively because of its simplicity. However, it has a drawback that the current at the feed segment depends on the length of the divided segments. In order to eliminate the problem, Tsai proposed a magnetic frill source model [5], which includes some computational difficulties in evaluating the near field of the magnetic frill, and then derived a simple expression of E_z which can be employed in analysis of parallel monopoles such as the Yagi-Uda and log-periodic antennas [6]. Junker presented a source model in which the incident electric field has a Gaussian distribution [7]. The model is simple but the length of the driving gap can not be involved in the source model since the length of the gap is determined only by the radius of the wire rather than physical gap length. Werner introduced a constant gap source model for the application to the MoM analysis with full kernel formulation, and compared the results with the experimental data [8]. The length of the constant gap is exactly the same to the real geometry. However, the point matching technique is used in the analysis, which usually causes the problem that the evaluated current distribution varies with the position of the matching points. In this letter, a new source model for MoM analysis by using sinusoidal reaction matching is proposed. The analysis results are compared with the results of the conventional methods and measurement to show the accuracy of the proposed source model.

2. Theory

Figure 1(a) shows the geometry of the analysis model, which is a cylindrical dipole antenna. The length and the radius of the cylindrical dipole is $2h$ and a, respectively, and the length of the feed gap is b. Conventionally, the dipole antenna is divided into dipole segments having the same length as shown in Fig. 1(b). In this case, the length of the driving segment has no relation with the length of the gap b and this model can not ac-

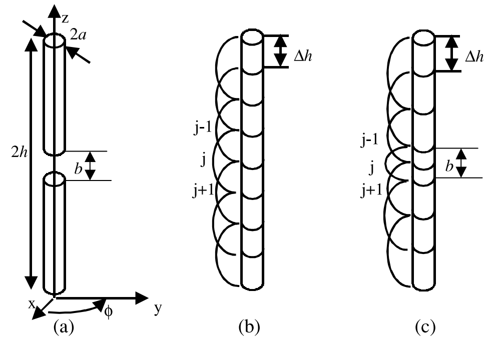

Fig. 1 Geometry of the cylindrical dipole antenna (a) and its analysis models (b) and (c).

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count for effect of the feeding gap exactly. In this letter, it is assumed that the source model has a fixed length that is equal to the length of the driving gap, which is shown in Fig. 1(c), and the incident field distribution is uniform in the driving gap, i.e.

$$E_{inc} = \begin{cases} 1 & (z_{j+1} \geq z \geq z_{j-1}) \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where j is the feeding segment number. Since the basis functions and the test functions of the sinusoidal reaction matching technique are given by

$$\mathbf{f}_i(z) = \begin{cases} \dfrac{\sin k(z_{i+1} - z)}{\sin k(z_{i+1} - z_i)}\hat{\mathbf{z}}, & z_{i+1} \geq z \geq z_i \\ \dfrac{\sin k(z - z_{i-1})}{\sin k(z_i - z_{i-1})}\hat{\mathbf{z}}, & z_i \geq z \geq z_{i-1}, \end{cases} \tag{2}$$

the elements in the voltage vector are evaluated as

$$\begin{aligned} V_i &= \int_{z_{i-1}}^{z_{i+1}} \mathbf{f}_i(z) \cdot \mathbf{E}_{inc}(z) dz \\ &= \begin{cases} \dfrac{2}{k \sin \dfrac{kb}{2}}(1 - \cos \dfrac{kb}{2}), & (i = j) \\ \dfrac{1}{k \sin \dfrac{kb}{2}}(1 - \cos \dfrac{kb}{2}), & (i = j \pm 1) \\ 0 & \text{otherwise,} \end{cases} \end{aligned} \tag{3}$$

and the input impedance of the antenna is evaluated by

$$Z_{in} = \frac{V_{j-1} + V_j + V_{j+1}}{I_j}, \tag{4}$$

where I_j is the current at the feeding segment.

In MoM analysis for thin wire antennas, the reduced-kernel (thin wire approximation) is usually used when the radius of the segment is small enough compared with the wavelength and the length of the dipole segment. However, since the length of the driving segment is fixed to b, the reduced-kernel approximation could lead to inaccurate results if radius a is too large to be negligible compared with b. In this case, the full kernel formulation should be applied instead of the reduced kernel formulation. The mutual impedance Z_{ij} of the wire dipole segment with radius of a is calculated by

$$Z_{ij} = \frac{1}{2\pi} \int_0^{2\pi} Z_{ij}^t(r = a\sqrt{2(1 - \cos \phi)})d\phi, \tag{5}$$

where Z_{ij}^t is the mutual impedance between two infinitely thin wire dipole segment separated with a distance of r by using reduced kernel.

Table 1 Definition of the source models.

Incident field distribution	Length of driving segment	
	Variable length	Constant length
Delta function	**Delta/Variable**	**Delta/Constant**
Pulse function	—	**Pulse/Constant**

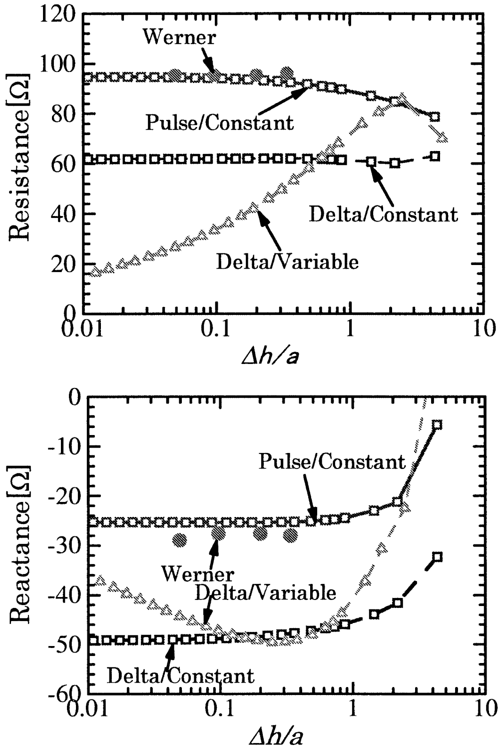

Fig. 2 Input impedance versus h/a for a half wavelength dipole antenna ($h = 0.25\,\lambda$) of radius $a = 0.0509\,\lambda$ with a ratio of length of driving gap to radius b/a is 1.189.

3. Results

The input impedance of a half wavelength cylindrical dipole antenna is calculated by using MoM with different source models, which include "Delta/Variable" model, "Delta/Constant" model, and "Pulse/Constant" model, shown in Table 1. The "Delta/Variable" model is usually called "delta gap model," where the incident electric field has a delta function distribution and the length of the feeding segment has the same length as the other segments that depends on the division number. The distribution of the incident filed of the "Delta/Constant" model is also assumed to be a delta function, while the length of the gap is kept to be the same to the driving gap. The "Pulse/Constant" model is defined by Eq. (3). The sinusoidal reaction matching technique and full kernel formulation are applied in the numerical analysis.

The numerical results of these models are compared with those given by Werner [8]. The input impedance versus the ratio of the length to the radius of divided dipole segment is shown in Fig. 2. The solution of the "Delta/Variable" diverges when $\Delta h/a$ becomes small. The solution of the "Delta/Constant" converges to a value different from Werner's solution.

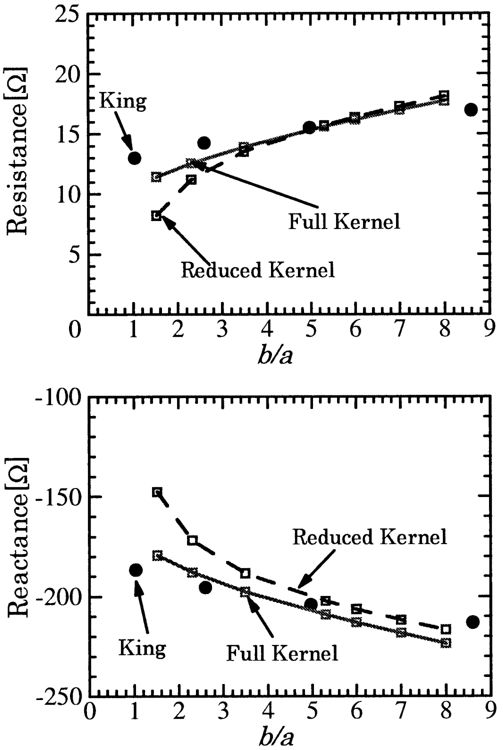

Fig. 3 Input impedance versus b/a for a quarter wavelength dipole antenna ($h = 0.125\,\lambda$) of radius $a = 0.009525\,\lambda$ when the dipole is divided into 11 dipole segments.

However, the “Pulse/Constant” yields a converged solution, which agrees with Werner’s solution very well. Since the moment method by using sinusoidal reaction matching technique is one of the Galerking’s methods, the solution from this method is more stable and more accurate than Werner’s solution where the point matching is applied.

The input impedance versus b/a for a quarter wavelength dipole antenna with radius $a = 0.009525\,\lambda$ is shown in Fig. 3 to investigate whether the proposed source model can incorporate the effect of the feeding gap or not. The results by using the “Full Kernel” formulation and the “Reduced Kernel” approximation are shown and compared to find the limitation of the reduced kernel approximation, and the results by King [9] are also plotted to confirm vardility of the computation. Since the model in [9] is a cylindrical monopole driven by a coaxial line which is different from the cylindrical dipole of this study, shown in Fig. 1(a), the gap width b of the dipole is assumed to be equivalent to the difference of outer and inner diameters of the coaxial lines approximately. However, it is found that the solution of the proposed source model follows the tendency of the King’s data well. It is also demonstrated that the reduced kernel is only effective when b/a is more than about 5, while the full kernel formulation is valid even if b/a is small.

4. Conclusions

A new source model for MoM analysis by using sinusoidal reaction matching technique has been proposed. The numerical results of the new source model have been compared with the conventional results. It has been demonstrated that the new model incorporates the effect of length of the feed gap well and the solution from the new source model is accurate and stable. The proposed source model is simple and easy to use. This source model, together with the full kernel formulation, can be applied to analyze the linear dipole antennas with no limitation of ratio of segment length to radius.

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