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LETTER

Impedance Analysis of Printed Antenna on Three-Dimensional High-Permittivity Dielectric Substrate Using Mixed-Domain MoM

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SUMMARY An integral equation approach with a new solution procedure using moment method (MoM) is applied for the computation of coupled currents on the surface of a printed dipole antenna and inside its high-permittivity three-dimensional dielectric substrate. The main purpose of this study is to validate the accuracy and reliability of the previously proposed MoM procedure by authors for the solution of a coupled volume-surface integral equations system. In continuation of the recent works of authors, a mixed-domain MoM expansion using Legendre polynomial basis function and cubic geometric modeling are adopted to solve the tensor-volume integral equation. In mixed-domain MoM, a combination of entire-domain and sub-domain basis functions, including three-dimensional Legnedre polynomial basis functions with different degrees is utilized for field expansion inside dielectric substrate. In addition, the conventional Rao-Wilton-Glisson (RWG) basis function is employed for electric current expansion over the printed structure. The accuracy of the proposed approach is verified through a comparison with the MoM solutions based on the spectral domain Green's function for infinitely large substrate and the results of FDTD method.

key words: *volume integral equation, mixed-domain MoM, dielectric substrate, printed antenna, legendre expansion, FDTD method*

1. Introduction

The study of the electromagnetic wave scattering from arbitrarily shaped three-dimensional dielectric bodies has been of considerable interest. This is mainly due to its broad range of applications such as analysis of printed antennas on finite size substrate made of dielectric materials. These studies have resulted in analytical and full-wave numerical methods which have been widely used in designs of various types of printed and patch antennas. Typical numerical methods for a printed structure analysis include method of moments (MoM), finite-difference time-domain method (FDTD), finite elements method (FEM), and the transmission line method (TLM).

Among these techniques, those that are based on integral equations, usually rely on the layered Green's function using Sommerfeld formulation, and hence can only be applied to infinitely large and mostly flat dielectric substrates. In practical applications, these approaches yield acceptable results whenever the substrate size is much larger than that of the printed structure. However, it is known that when the substrate edge is close to the printed antenna, the input impedance of the patch antenna can not be estimated using integral equation approach with Green's function for infinitely large dielectric substrate. In these cases, FDTD method is also inadequate and does not yield a desirable accuracy for substrates made from high-permittivity materials.

In this work, we treat the dielectric substrate as a three-dimensional body and hence, a coupled tensor-volume/surface integral equations (TVSIE) system is adopted for fields calculations. In continuation of our previous works [1]–[3], we have solved these integral equations by using the Galerkin's MoM. In the MoM procedure, we have adopted hybrid-domain Legendre polynomial expansion for the three-dimensional dielectric substrate, and used Rao-Wilton-Glisson (RWG) based function for the printed antenna.

There are some reports of similar studies for modeling of printed structures with finite size substrates by MoM using various basis and testing functions, numerical integration and geometric modeling [4]–[6].

Nevertheless, there are few differences between this work and previous studies. The major distinction is the expansion domain and its corresponding basis functions adopted for MoM procedure. It is based on so-called mixed-domain or hybrid domain moment method [1], [3], that applies three-dimensional Legendre polynomials with various degrees for field estimation inside the dielectric body, using a combined sub-domain and entire-domain expansion methods.

In our work, we have essentially benefited from the compromise that such hybrid expansion provides between accuracy and efficiency. Hybrid expansion combines the entire-domain and sub-domain by using polynomial functions with various degrees. It also uses a set of orthogonal basis functions that helps avoid an ill-condition MoM matrix in the modeling. Further information on these can be found in [1]–[3].

In order to evaluate the accuracy of the proposed method, we have compared its numerical results with those of MoM-based on spectral Green's functions for infinitely large substrate, as well as FDTD method with very fine cells.

2. Coupled Sub-Domain Surface and Mixed-Domain Volume Expansions in MoM

Input impedance of a printed antenna over a finite size dielectric substrate can be numerically calculated by MoM

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for a system of coupled TVSIE. The basic idea behind the TVSIE is to utilize the surface and volume equivalent electromagnetic theorems to replace a conducting antenna with a surface current and a dielectric volume with a polarization current. This results in a set of coupled integral equations to be solved simultaneously. This in turn, yields the total field setup by the radiator/dielectric system, and the circuit characteristics of the radiator itself. This system of coupled TVSIE is given by

$$\begin{pmatrix} \frac{1}{j\omega\Delta\varepsilon} + L_D & L_A \\ (\hat{n} \times L_D) & (\hat{n} \times L_A) \end{pmatrix} \begin{pmatrix} \boldsymbol{J}_p \\ \boldsymbol{J}_a \end{pmatrix} = \begin{pmatrix} \boldsymbol{E}^{\text{inc}}(r \in V) \\ \hat{n} \times \boldsymbol{E}^{\text{inc}}(r \in S) \end{pmatrix} \tag{1}$$

where

$$\begin{aligned} L_D(*) &= j\omega\mu_0 \int\int_V\int \overline{\overline{G_0}}(\boldsymbol{r}, \boldsymbol{r'}) \cdot (*)\, dv' \\ L_A(*) &= j\omega\mu_0 \int\int_S \overline{\overline{G_0}}(\boldsymbol{r}, \boldsymbol{r'}) \cdot (*)\, ds' \\ \Delta\varepsilon &= \varepsilon(\boldsymbol{r}) - \varepsilon_0 \end{aligned} \tag{2}$$

and $\hat{n}$ and $\overline{\overline{G_0}}$ are surface normal vector over the printed structure and free-space dyadic Green's function, respectively. The Next step is to discretize and solve the above equations by MoM. The choice of expansion functions used to represent the unknown currents or fields in formulation of a radiation or scattering problem, can greatly affect the accuracy and convergence of MoM solutions. In this work, we have applied RWG basis function [7] to estimate current on the surface of the conducting printed structure. RWG basis functions give a high level of accuracy for current distribution on conducting surfaces.

Furthermore, we have expanded the field inside the dielectric body using a mixture of three-dimensional orthogonal polynomials with various degrees. Indeed, the main profile of the modal field is well represented by the entire-domain basis function and fewer sub-domain basis functions are required for further refinement of the field representation. This improves the accuracy compared to formulation using only sub-domain basis function. On the other hand, a set of orthogonal basis function such as Legendre polynomials allows the use of higher-order basis functions without introducing an ill-condition MoM matrix. Details have been extensively discussed in [1]–[3]. Ultimately, the adopted three-dimensional polynomial basis function is expressed as follows

$$\begin{aligned} \boldsymbol{J}_{\mathrm{P}}(\boldsymbol{r}) &= \sum_{n=1}^{N}\sum_{c=1}^{3}\sum_{i=0}^{N_{nx}}\sum_{j=0}^{N_{ny}}\sum_{k=0}^{N_{nz}} a_{ijk}^{(nc)} \boldsymbol{g}_{ijk}^{(nc)}(\boldsymbol{r}) \\ \boldsymbol{g}_{ijk}^{(nc)}(\boldsymbol{r}) &= P_i\left(\frac{x - x_n}{\ell_{xn}}\right) P_j\left(\frac{y - y_n}{\ell_{yn}}\right) P_k\left(\frac{z - z_n}{\ell_{zn}}\right) u_n(\boldsymbol{r})\hat{c} \\ (\hat{1}, \hat{2}, \hat{3}) &= (\hat{x}, \hat{y}, \hat{z}) \\ u_n(\boldsymbol{r}) &= \begin{cases} 1, & \boldsymbol{r} \in V_n \\ 0, & \text{Otherwise} \end{cases} \\ V_n &= (x_n - \ell_{xn}, x_n + \ell_{xn}) \times (y_n - \ell_{yn}, y_n + \ell_{yn}) \\ &\quad \times (z_n - \ell_{zn}, z_n + \ell_{zn}), \\ &2\ell_{xn} \times 2\ell_{yn} \times 2\ell_{zn}: \text{ Cube Size} \\ &(x_n, y_n, z_n): \text{ Cube Center} \end{aligned} \tag{3}$$

where P_i is the Legendre polynomial of the ith order.

Substitution of basis functions into the system of integral Eq. (1), and applying the inner product procedure, yields a simultaneous algebraic equations system. Mathematical details are previously discussed in [1].

There are three types of singularity in the inner product integrals in Galerkin's method. In computation of volume elements of impedance matrix of mixed-domain MoM, singularity happens in the calculation of generalized self-impedance terms. In addition, it appears in the calculation of some mutual-impedance elements between blocks of which integration regions overlap to improve current continuity condition. Detailed analysis of removing such singularities was presented previously in [1].

In the case of surface-surface elements, the singular integrals are evaluated using the classical method of RWG basis function [7]. Finally, in the case of volume-surface terms, when the surface of the printed structure coincides with the surface of a volume cell, singularity occurs in the corresponding inner product integral. We have applied Duffy method [6], [8] to treat these singular conditions. In Duffy's approach, the domain, which includes a singular point, is first subdivided into certain sub-regions that share a common vertex at the singular point. Next, each sub-region is mapped to a unit domain via a change of variable. It can be shown that the integrand in the new domain is not singular and regular Gaussian quadrature rules can be applied to evaluate the integral [8].

3. Numerical Results

In this section, we present a few numerical examples to demonstrate the application of the proposed solution of TVSIE in impedance computation of a printed dipole antenna with a three-dimensional substrate. We use the model in Figure 1 for analysis purpose. The following shows the model dimensions:

$L_z = L_x = 200$ mm, $L_y = 2.4$ mm, $w = 3$ mm, $2H = 100$ mm. In the MoM procedure, the dielectric material is first discreteized into $20 \times 3 \times 20$ blocks where the size of blocks is equally $10\,\text{mm} \times 0.8\,\text{mm} \times 10\,\text{mm}$. Next, for the hybrid-domain computation, the whole dielectric body is considered as an entire-domain region with the corresponding polynomial degree of $N_{nx} = N_{ny} = N_{nz} = 5$ in (3). Only the sub-domain blocks on the sides of the entire-domain volume are kept for overlapping regions with the polynomial degree of $N_{nx} = N_{ny} = N_{nz} = 1$. The printed antenna is divided into 10 segments yielding 20 triangles for RWG elements [7]. It should also be mentioned that printed antenna is assumed to be infinitely thin. The feed model is delta-gap which is ideally suited for RWG edge elements. This gap is associated with an inner edge which is corresponding to one

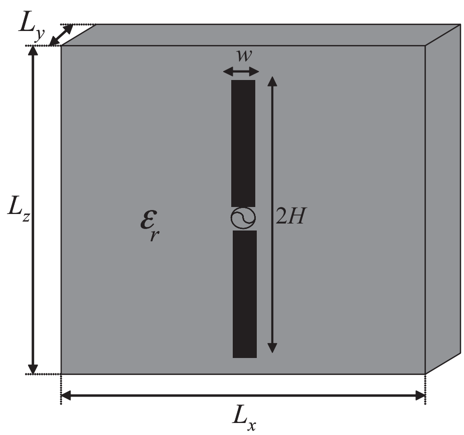

Fig. 1 Analysis model of printed dipole antenna on finite substrate.

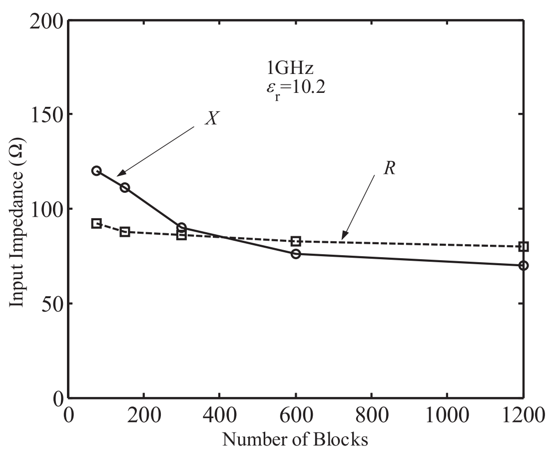

Fig. 2 Input impedance vs. number of initial blocks for $\varepsilon_r = 10.2$.

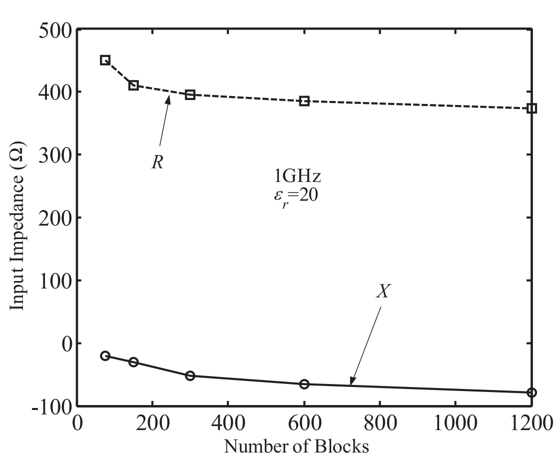

Fig. 3 Input impedance vs. number of initial blocks for $\varepsilon_r = 20$.

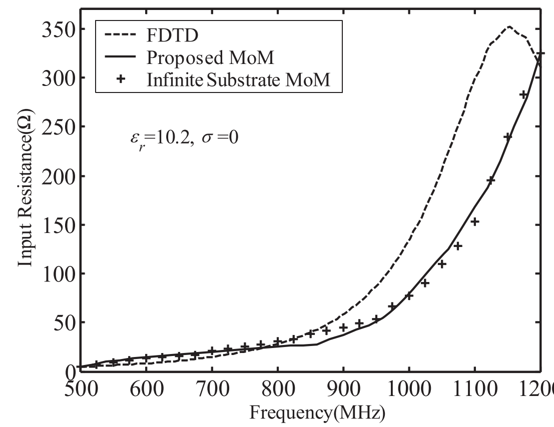

Fig. 4 Input resistance.

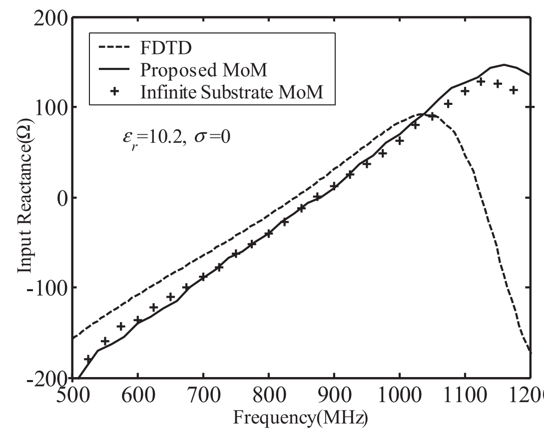

Fig. 5 Input reactance.

RWG element in the middle of the printed conducting strip [7].

In FDTD calculations, number of cells is 881 × 107 × 441 and the number of time steps is taken as 30,000.

Also, the cell dimensions are $\Delta x = 0.25$ mm, $\Delta y = 0.4$ mm, and $\Delta z = 0.5$ mm. It should be mentioned that the adopted cell size is quite small compared with the effective wavelength of the operating frequency band. This was primarily done to obtain results with highest possible accuracy by FDTD technique for better comparison with the MoM results. FDTD calculations were performed on an SX-7 supercomputer at Tohoku University.

Figures 2 and 3 show the input impedance of the printed dipole at two different permittivities versus the number of blocks during the initial sub-domain discretization using side-overlapping procedure [1] in mixed-domain MoM at 1 GHz. The results show reasonable convergence rate of the proposed MoM.

Figures 4 and 5 indicate the input impedance of the printed dipole for $\varepsilon_r = 10.2$ and Fig. 6 and 7 show it for $\varepsilon_r = 20$.

It is worth mentioning that the proposed method is also able to handle lossy dielectric substrate as it has been shown in [1]–[3].

The accuracy of the proposed MoM approach can be verified by looking at its agreement of its results with the results of MoM based on the spectral Green's functions for infinitely large substrate [9]. It is observed that for higher-permittivity substrate, the difference between the MoM results and the FDTD method results is greater than that of

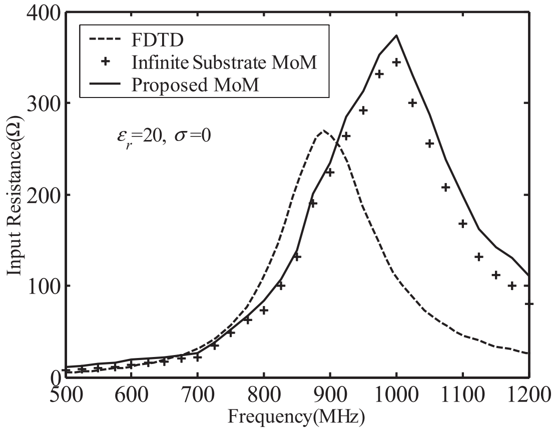

Fig. 6 Input resistance.

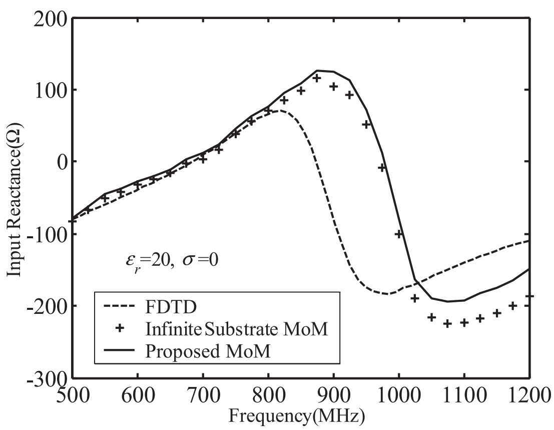

Fig. 7 Input reactance.

lower-permittivity case, which confirms the main drawback of the FDTD method.

4. Concluding Remarks

A new solution procedure using mixed-domain Galerkin-based MoM was applied to achieve an accurate and efficient solution of a coupled tensor-volume/surface integral equation for calculation of the input impedance of a printed dipole antenna over a finite size dielectric substrate. This procedure was carried out using three-dimensional Legendre polynomial expansion in combination with RWG basis function. The results were contrasted with those of FDTD method and spectral Green's function-based MoM for infinitely large substrate. This comparison constituted our accuracy analysis of the proposed method for relatively high values of permittivities. Further improvement in accuracy can be achieved by satisfying the continuity of the normal component of fields across the elements boundaries in subsectioning procedure.

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