Abstract—The effect of the mutual coupling between the array elements on the performance of the adaptive array antennas (AAA) is investigated when the actual received voltages which include the mutual coupling are directly used to estimate the weight vector based on the adaptive algorithm. The output signal-to-interference-noise ratio (SINR), the convergence of the adaptive algorithm and the synthesized pattern are evaluated to study the effect due to the existence of the mutual coupling. It is found that the mutual coupling affects the antenna adaptive gain, but does not affect the adaptive processing. It is also found that the mutual coupling does not always degrade the iterative convergence of the adaptive algorithm. It is proved that any invertible matrix for compensating the mutual coupling cannot improve the output SINR. It is also indicated that the radiation pattern can be correctly synthesized in the presence of the mutual coupling by introducing the universal steering vector (USV) whose element corresponds to the array element pattern.

Index Terms—Adaptive array, antenna, array element pattern, array factor, method of moments (MoM), mutual coupling, steering vector.

I. INTRODUCTION

Since adaptive array antennas are capable of reducing multipath fading of the desired signal and suppressing the interferences, the use of adaptive array antennas is one of the most effective ways of increasing the frequency efficiency and improving the communication quality in the mobile communication system [1], [2]. Although adaptive antennas have been mainly investigated for implementation in base stations, a great deal of attention has turned to evaluating their performance when mounted on mobile handsets [3]-[6]. Because there is not sufficient space to implement array antennas with large array spacing on the handsets, the mutual coupling between array elements are usually strong and should be dealt with correctly in the adaptive signal process.

It has been pointed out that the performance of an adaptive array antenna is affected by the mutual coupling and this effect, which is particularly serious for small array spacing, decreases the output signal-to-interference-noise ratio (SINR) and degrades the convergence of the least mean square (LMS) [7]. It has also been pointed out that the synthesized pattern of an adaptive array antenna cannot be evaluated correctly because the weight vector is incorrectly obtained by the received voltages at the presence of the mutual coupling so that the synthesized pattern cannot direct the main beam to the desired wave and nulls to the interference directions [8]-[10]. Therefore, many approaches have been developed to compensate the mutual coupling. These approaches include the open-circuit voltage method for the estimation of direction of arrival (DOA) [11], [12], the method combined with moment method for adaptive control [8], [13], and the approach using the redefined impedance matrix for beam forming [9], [10]. The purpose of the previous researches was to remove the mutual coupling from the received voltages. And then the compensated voltage was used to compute the weight vector by using the adaptive algorithm, or to compute the spectrum versus directions and the synthesized pattern by using the DOA estimation algorithm.

In this study, because the output SINR is increased by the gain obtained in the adaptive processing based on the input SINR, both the output SINR and the input SINR are investigated in the presence of the mutual coupling to understand which is fundamentally affected by the mutual coupling. The effect of the invertible compensation matrix for compensating the mutual coupling on the output SINR and the convergence of least mean squared (LMS) optimum algorithm with the presence of the mutual coupling are also studied. Further, the synthesized pattern is evaluated by using the so called the universal steering vector (USV) instead of the conventional steering vector (CSV) because we believe that the incorrect synthesized pattern is caused by the use of the CSV which does not include the mutual coupling. The USV whose elements are equal to the array element pattern includes the mutual coupling and has been successfully applied to the DOA estimation using the array antenna with arbitrary geometry [14].

Section II presents the numerical approach to simulate the received voltages with and without the mutual coupling and the method to calculate the USV. Section III demonstrates the input and output SINR of 6-element dipole array antenna to investigate how the mutual coupling affects these two parameters. Section IV shows the effect of an invertible compensation matrix on the output SINR and Section V shows the convergence behaviors of the LMS algorithm. Finally, the synthesized patterns of the 6-element dipole array antenna and a 2-element monopole array antenna mounted on a mobile handset are illustrated in Section VI to verify the effeteness of the USV.
II. RECEIVED VOLTAGE VECTOR AND UNIVERSAL STEERING VECTOR

A. Received Voltage Vector With and Without Mutual Coupling Effect

Assuming that an $M$-element linear antenna array is used as the receiving antenna illuminated by $L$ incident waves. The received voltage vector $[V]$ with size of $M$ in the presence of the mutual coupling can be expressed as

$$[V] = \sum_{j=1}^{L} [A_p^m(\theta_j, \phi_j)] E_p^{inc}(\theta_j, \phi_j) + [V^n] \quad (1)$$

where $E_p^{inc}(\theta_j, \phi_j)$ denotes the electric field of incident wave from $(\theta_j, \phi_j)$ with polarization represented as $p$. $[V^n]$ is the voltage vector with dimension $M$ caused by the thermal noise. $[A_p^m(\theta_j, \phi_j)]$ represents the USV whose elements are equivalent to the array element pattern.

The $[V^{uw/\phi}]$ expressed in the following equation represents the received voltages without the mutual coupling effect

$$[V^{uw/\phi}] = \sum_{j=1}^{L} [A_p^m(\theta_j, \phi_j)] E_p^{inc}(\theta_j, \phi_j) + [V^n] \quad (2)$$

where $[A_p^m(\theta_j, \phi_j)]$ represents the CSV and is equivalent to the array factor which does not include the mutual coupling between array elements.

B. USV Calculation

The USV can be obtained by measuring the actual complex array element pattern of each element directly. However, the accurate measurement of the array element pattern is complicated and a huge memory is needed because the element pattern of each array element for all direction is required to be stored [15]–[19]. Therefore, a numerical method of on-site evaluation of the USV is required. An efficient method to evaluate the USV for $M$-element array antenna with arbitrary geometry by using method of moments (MoM) is presented in the following.

In this research, the subdomain MoM analysis is used for calculating the USV [20]. The $i$th element of the array is divided into $N_i$ dipole segments. The segment lengths are required to be less than $\lambda/6$. The total number of the dipole segments is $N(= \sum_{i=1}^{M} N_i)$. Assuming that all the array elements are terminated by an impedance of $Z_i$, the received voltage including the effect of the mutual coupling between the array elements is given by

$$[A^n(\theta, \phi)] = Z_i [V^{\text{ext}}] [V^{\text{IRF}}(\theta, \phi)] \quad (3)$$

where $[V^{\text{ext}}]$ with dimension of $M \times N$ is the admittance matrix which includes the mutual admittance between the segments on the terminals and all the segments of the array elements. This admittance matrix can be calculated by inversing the impedance matrix which includes the mutual impedance between all the segments of the array elements and extracting the corresponding part from the inversed matrix. $[V^{\text{IRF}}(\theta, \phi)]$ is the $N$ voltage vector representing the inner products of the weighting functions of all the segments and the incident electric field from direction $(\theta, \phi)$. The USV of the antenna array expressed in (3) can be applied to the array elements with arbitrary geometry [14].

III. INPUT SINR AND OUTPUT SINR IN THE PRESENCE OF MUTUAL COUPLING

The received voltage vectors in (1) and (2) can be expressed as

$$[V] = [V^d] + [V^u] + [V^n] \quad (4)$$

where $[V^d]$ denotes the voltage caused by one desired wave, $[V^u]$ denotes the voltage caused by $(L - 1)$ undesired waves. In the wireless communications, usually several coherent desired incident waves should be considered due to the reflection. However, in this study, in order to investigate whether the radiation pattern can be synthesized accurately or not, the adaptive algorithm is applied to the situation of the incidence waves including one desired wave and several incoherent interference waves. The assumption is equivalent to the practical situation where only the maximum peak of the desired waves together with the interference waves is captured.

If all incident waves are incoherent, the input SINR of $M$-element array antenna, defined at the input port of the adaptive processing after the reception of antennas, can be given by

$$\text{SINR}_{\text{in}} = \frac{\sum_{i=1}^{M} |v^d_i|^2}{\sum_{i=1}^{M} |v^u_i|^2 + \sum_{i=1}^{M} |v^n_i|^2} \quad (5)$$

where $v^d_i$, $v^u_i$, and $v^n_i$ are elements of $[V^d]$, $[V^u]$, and $[V^n]$ respectively.

The output SINR which is defined at the output port of the adaptive processing can be obtained by the following:

$$\text{SINR}_{\text{out}} = \frac{\left| \sum_{i=1}^{M} u^d_i v^d_i \right|^2}{\left| \sum_{i=1}^{M} u^u_i v^u_i \right|^2 + \left| \sum_{i=1}^{M} u^n_i v^n_i \right|^2} \quad (6)$$

where vector $[W]$ represents the weight vector which can be calculated to optimize some properties of the received signal such as the maximum output signal of array in terms of the minimum-mean-squared error (MMSE), or the maximum SINR.

In the MMSE adaptive array system, the optimum weight vector $[W]$ in the steady state can be estimated as

$$[W] = [R_{xy}]^{-1} [r_{yv}] \quad (7)$$

where $[R_{xy}]$ and $[r_{yv}]$ are the covariance matrix of the received voltages and the reference correlation vector, respectively, and given by

$$[R_{xy}] = E ([V][V]^H)$$

$$[r_{yv}] = E (r(t)[V]^{*}) \quad (8)$$

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where \( E() \) denotes the statistical expectation over time because the received signal is a modulated time-varying signal. The superscript asterisk and \( H \) denote the complex conjugate and the transpose conjugate, respectively, and \( r(t) \) represents the reference signal.

In a practical adaptive antenna system, the optimum weights at time \( t \) can be obtained by the LMS algorithm through the following iteration equation [21]:

\[
[W(t)] = [W(t - \Delta t)] + \mu e^s(t) [V(t)]^H \tag{10}
\]

where \( \Delta t \) is the time interval between two iterative steps, \( \mu \) is the step size, \( e(t) \) is the error between the array output and the reference signal, and is calculated by

\[
e(t) = r(t) - [W(t)]^H [V(t)]. \tag{11}
\]

Usually, each antenna array has a different input SINR shown in (5) even with the same incident waves in the high thermal noise system. Therefore, another parameter called incident electrical field-to-noise ratio (ENR) is introduced in the research. \( E \) is the incident electric field before the reception of antennas, while \( N \) means the thermal noise after the reception of antennas. In a high thermal noise system which is assumed in this research, the ENR is regarded to be independent of the array antenna’s property and is defined as the following:

\[
\text{ENR} = \frac{|E_\text{inc}|^2}{P_n} \tag{12}
\]

where \( P_n \) denotes the thermal noise power of the antenna system, and incident ENR has a unit of \( \text{dB} \Omega/\text{m}^2 \). It should be noted that the definition of ENR in (12) is used for expressing the desired wave. The other interference waves are expressed by the parameter SIR (signal-to-interference), which gives the relative strength to the desired wave. Therefore, the parameter ENR, together with the parameter SIR, is used to express the incident waves.

Fig. 1 shows a 6-element dipole array for numerical simulation. The length of dipole element is \( l = 0.5\lambda \), and each element of the array is loaded by 50 \( \Omega \). The input and output SINR of this array antenna with different element spacing \( d \) are compared in Fig. 2. The abscissa of Fig. 2 stands for the spacing \( d \) between the elements and vertical axis denotes the SINR. The two cases with interference and without interference are considered in Fig. 2. In the case without interference, only one desired wave with the incident ENR of 40 \( \text{dB} \Omega/\text{m}^2 \) in the desired direction is included, while in the case with interference, the undesired wave with SIR of \(-10 \text{ dB} \) from direction of \( \phi_\text{nu} = 30^\circ \) is added to the desired wave.

In Fig. 2, it is found that both of the output and the input SINR decrease when the element spacing becomes small. When the interference is not included in the incident, the difference between the output SINR and the input SINR is kept to 7.8 dB which is equivalent to \( 10 \log(M) \). When the interference is considered, the input SINR is decreased due to the interference, but the output SINR remains approximately the same. The results demonstrate the mutual coupling degrades the input SINR and the output SINR by decreasing the antenna gain, but does not affect operation of the adaptive processing.

IV. EFFECT OF INVERTIBLE COMPENSATION MATRIX ON OUTPUT SINR

From Fig. 2, it has been found that the output SINR is decreased when the array spacing \( d \) becomes small. Here, the research investigates whether the output SINR can be improved by implementing an invertible matrix \( [T] \) to remove the mutual coupling effect from the received voltage vector \( [V] \) before the adaptive processing. The invertible matrix \( [T] \) is inserted between the array terminals and the adaptive processor as shown in Fig. 3. The compensated signals \( [X] \) can be obtained by

\[
[X] = [T][V] \tag{13}
\]

The steady state weight vector \( [U] \) in MMSE adaptive array system is obtained similarly to (7) as

\[
[U] = [R_{ux}]^{-1}[r_{ux}] \tag{14}
\]

where \( [R_{ux}] \) is the covariance matrix of the compensated voltages, \( [r_{ux}] \) is the reference correlation vector, and given by

\[
[R_{ux}] = E([X][X]^H) \tag{15}
\]

\[
[r_{ux}] = E(r(t)[X]^s). \tag{16}
\]
The output $S_{\nu}$ with compensation and the output $S_{\nu}$ without compensation can be obtained by the following:

$$S_{\nu} = [W]^H[V],$$  \hspace{1cm} (17)

$$S_{\nu} = [U]^H[X].$$  \hspace{1cm} (18)

Since the matrix $[T]^H$ is invertible, the weight vector $[U]$ in (14) can be obtained by

\nu}]^{-1}[T]^{-1}.$$  \hspace{1cm} (19)

Therefore, substituting (13) and (19) into (18), the following is obtained:

$$S_{\nu} = [r_{\nu
\nu}]^H[R_{\nu
\nu}]^{-1}[V].$$  \hspace{1cm} (20)

The right side of the above equation is equal to $S_{\nu}$ of (17), indicating that inserting an invertible compensation matrix $[T]$ between the array terminal and the adaptive processor gives no effect on the output signal and cannot improve the output SINR.

V. CONVERGENCE OF LMS ALGORITHM WITH AND WITHOUT MUTUAL COUPLING

Since the convergence of LMS algorithm becomes fast when the ratio of the maximum eigenvalue to the minimum eigenvalue of the covariance matrix decreases, it is very important to investigate the eigenvalues of the covariance matrix before evaluating the convergence of the adaptive algorithm. For $M$-element array, there are $M$ eigenvalues which are associated with the power of the desired signal, the power of the undesired signals, and the noise power. When the undesired signals are absent, the maximum eigenvalue can be given by [22]

$$\lambda_1 = \lambda_{\text{max}} = \sigma^2 (1 + \text{SINR}^\text{out}),$$  \hspace{1cm} (21)

where, the remaining eigenvalues $\lambda_i = \sigma^2 (i = 2, \cdots, M)$ associated with the noise power $\sigma^2$ are equal.

Fig. 4 shows the convergence of the mean squared error (MSE) of LMS algorithm for the 6-element dipole array antenna versus the element spacing $d$ with the mutual coupling effect and without the mutual coupling effect. Fig. 5 shows their corresponding eigenvalues. In the simulation, the DOA of the desired signal is $(\theta = 90^\circ, \phi = 0^\circ)$ and the incident ENR is assumed to be 40 dB. The convergence becomes fast even though the mutual coupling becomes stronger when $d$ is reduced from 0.5 $\lambda$ to 0.1 $\lambda$. The reason is that the maximum eigenvalue becomes small when the element spacing $d$ is reduced while the minimum eigenvalue is approximately the same as the array antenna with large element spacing $d$.

The output SINR versus incident angle for different element distance is shown in Fig. 6. The terminal impedance of each antenna element for all cases is set to be 50 $\Omega$. The results show that the output SINR changes with the element spacing $d$ and the incident angle. In the case of incident angle $\phi$ is equal to $0^\circ$, the output SINR increases when the element spacing $d$ becomes large. As a result, it makes the convergence of the LMS algorithm slow down as shown in Fig. 4. However it should be noted that in the above case the mutual coupling becomes weak when the element spacing $d$ becomes large. Therefore, the numerical results indicate that the mutual coupling does affect the

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Fig. 3. Received voltages compensated by an invertible matrix [7].

Fig. 4. Convergence of LMS for 6-element dipole array.

Fig. 5. Eigenvalues of covariance matrix of the 6-element dipole array.
Fig. 6. Output SINR versus incident angle for various array element spacing $d$. 

Fig. 7. Synthesized pattern of 6-element dipole array. 

convergence of the LMS algorithm, but does not always degrade the convergence.

VI. SYNTHESIZED PATTERN BY USV AND CSV

The synthesized radiation pattern $f(\theta, \phi, p)$ of the array is calculated by

$$f(\theta, \phi, p) = [W]^H [A_w(\theta, \phi)]. \tag{22}$$

The synthesized patterns can be evaluated by using the USV, the CSV, and the Chebyshev synthesis under null constraints. However, the USV method is different from the two other methods in using the array element pattern instead of the array factor as the steering vector. In this study, the USV is used as the steering vector $[A_w(\theta, \phi)]$ in (22), because the USV includes the effect of the mutual coupling. While the weight vector $[W]$ is evaluated by using the received voltage vector directly without any compensation.

The synthesized patterns for 6-element dipole array by the USV and CSV are illustrated in Fig. 7 where the desired wave comes from $\phi = 0^\circ$, and three undesired waves come from $\phi = 30^\circ, 300^\circ, 330^\circ$ in $xy$ plane, respectively. The incident ENR of desired wave is $40 \text{ dB}$ and the SIR of three undesired waves are $0 \text{ dB}, 0 \text{ dB}, -10 \text{ dB}$, respectively. In the synthesized pattern obtained by the USV, the main maximum beam is controlled correctly to the DOA of the desired wave, and the nulls with the level less than $-40 \text{ dB}$ are directed rightly to the three undesired wave directions. However, in the synthesized pattern obtained by the CSV, only the main maximum beam is directed to the DOA of the desired wave, the directions of the nulls are not directed to the DOAs of the three undesired waves correctly.

Fig. 8 shows another array antenna which consists of 2-element monopole array antenna mounted on the mobile handset. Two monopole antennas have the same length of $1/3\lambda$ and are separated by $0.5\lambda$. Each monopole element is terminated by $50 \Omega$. The mobile handset is modeled to a rectangular conducting plane with size of $2\lambda/3 \times \lambda$. In the above array structure, not only does the mutual coupling occur between the antenna elements themselves, but also between the antenna elements and the mobile handset. The synthesized patterns both by the USV and CSV are illustrated in Fig. 9 where the desired wave comes from $\phi = 0^\circ$, and the undesired wave comes from $\phi = 50^\circ$ in $xy$ plane. The incident ENR is $40 \text{ dB}$ and the SIR of the
undesired wave is 0 dB. It is found that the pattern can be accurately synthesized even in the presence of the mutual coupling by using the USV, but not by using the CSV.

VII. CONCLUSION

The effects of the mutual coupling between the array elements on the performance of the AAA such as the SINR, the convergence of adaptive algorithm and the synthesized patterns have been investigated when the actual received voltages including the mutual coupling is directly used to estimate the weights for the AAA system with high thermal noise. It has been found that the mutual coupling affects the antenna gain, but does not affect the adaptive processing. The mutual coupling does not always reduce the iterative convergence of the adaptive algorithm. It has been proved that any invertible matrix for compensating the mutual coupling is directly used to estimate the weights for the adaptive processing. The mutual coupling does not always reduce the iterative convergence of the adaptive algorithm. It has been proved that any invertible matrix for compensating the mutual coupling cannot improve the output SINR. It has also been indicated that the radiation pattern can be synthesized correctly even in the presence of the mutual coupling by introducing the USV whose element corresponds to the array element pattern.

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REFERENCES


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