Impedance of a Large Circular Loop Antenna in a Magnetoplasma

SHIGEO OHNUKI, MEMBER, IEEE, KUNIO SAWAYA, MEMBER, IEEE, AND SABURO ADACHI, FELLOW, IEEE

Abstract—The input impedance of a large circular loop antenna with arbitrary orientation in a cold magnetoplasma is calculated by using a transmission line theory. New impedance resonances for antennas of finite size in a magnetoplasma in the frequency region below and near the electron cyclotron frequency are indicated theoretically. The resonance peak of the impedance at the lower hybrid resonance frequency is also predicted to exist for arbitrarily oriented antennas of finite size. The experiments on the impedance of a large circular loop antenna are carried out for the cases of normal and parallel orientation of the magnetic field with respect to the plane of the loop immersed in a radio frequency-generated laboratory plasma. The newly predicted impedance resonances for the antenna of finite size are observed. It is also shown that the measured impedances agree fairly well with the calculated ones.

I. INTRODUCTION

LOOP ANTENNAS have been used in space plasma to observe VLF/ELF noise and emissions in the ionosphere. In laboratory plasma loop antennas have been used as Whistler wave radiators [1] and as ion cyclotron range of frequency (ICRF) wave launcher for RF heating in nuclear fusion experiments [2]. The loop antenna in plasma is one of the most commonly used antennas as dipole antennas. Antenna characteristics of the loop in a magnetoplasma are important for the purpose of various applications in space and laboratory plasma. Theoretical and experimental studies on a loop antenna in a magnetoplasma have been reported hitherto by several researchers [3]–[9]. The theoretical studies on the impedance of a loop antenna have concerned themselves with a small loop. The impedance of a small loop in a cold magnetoplasma has been analyzed by the electromotive force (EMF) and Poynting vector methods [6], [7]. However, the theoretical analyses are usually too complicated to be practically useful. There also have been no theoretical formulas applicable to loop antennas of finite size which are not necessarily small in terms of a wavelength.

When the loop is as large or larger than resonant size in the free space, the circular loop antenna is customarily analyzed by the Fourier series method [10], for example. Fourier series results are in excellent agreement with measurements even for a fairly large loop. Fourier series analysis has been applied to a loop antenna in an isotropic compressible plasma [11]. But it is not easy to apply this method to the loop antenna immersed in a magnetoplasma. Therefore, we apply the transmission line theory proposed by Adachi et al. [12] which was proven to be very useful for the impedance calculation of a cylindrical dipole immersed in a cold magnetoplasma.

In this paper the input impedance of a large circular loop antenna with arbitrary orientation in a cold magnetoplasma is calculated. The input impedance of the antenna with an arbitrary diameter is expressed by a simple formula in terms of the characteristic impedance and the propagation constant of the transmission line constituting the loop. New impedance resonances in the high frequency region for loop antennas of finite size in a magnetoplasma are discussed in connection with the propagation constant. The impedance characteristics in the lower frequency region near the lower hybrid resonance frequency are also discussed. Experiments for a large circular loop antenna are carried out for the static magnetic field normal and parallel to the loop immersed in an RF-generated laboratory plasma. The measured results are then presented to confirm the theory.

II. CALCULATION OF IMPEDANCE BY THE TRANSMISSION LINE THEORY

A circular loop antenna with radius $a$ and wire radius $\rho$ is immersed in a uniform cold magnetoplasma as shown in Fig. 1. In cases where a radiation resistance is less significant or can be estimated with some other means, the input impedance of the circular loop antenna in zeroth-order approximation can be obtained by applying the transmission line theory. We approximate the loop as a distributed-constant uniform transmission. Hence, we shall obtain a distributed capacitance of the transmission line when the electric charge $q$ per unit length is uniformly distributed along the periphery of the loop. The distributed capacitance between two segments of the symmetrical positions with respect to the $x$-axis is not constant and the loop antenna can be considered as a nonuniform transmission line. For simplicity, the capacitance is typified by the value between the particular two points $P(0, a, \rho)$ and $P'(0, -a, -\rho)$ in Cartesian coordinates, that is, the capacitance at the midpoint of the semicircular arcs of the loop.

The electrostatic potential $\psi_p$ at the point $(x = 0, y = a, z = \rho)$ on the surface of the loop is obtained by solving the modified Poisson's equation as follows:

$$\psi_p = \frac{q}{4\pi\varepsilon_0\sqrt{K'K}G(\alpha, \theta, \phi)} / I(\alpha, \theta, \phi),$$

where

$$I(\alpha, \theta, \phi) = \int_{\sqrt{2}}^{\pi/2} \left( \frac{1}{\sqrt{R(\eta)}} + \frac{1}{\sqrt{R(-\eta)}} \right) \, d\eta,$$

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The distributed inductance $L$ for $\rho/a \ll 1$ may be obtained by the average self-inductance per unit length of one turn circular loop with uniform surface current in free space, since the plasma medium can be replaced by free space in magnetostatic sense, i.e.,

$$L = (\mu_0/\pi) \ln (8a/\rho).$$

By using both (6) and (7) we can obtain the characteristic impedance and propagation constant of the transmission line constituting the loop. However, as (6) is too complicated to give a practically useful and simple formula, the integral $I(\alpha, \theta, \phi)$ is replaced by the corresponding value for the non-magnetized plasma in this approximate analysis (Appendix). Substituting $\alpha = 1$ in (4) leads to

$$R(\eta) = R(-\eta) \sim 1 - \sin^2 \eta/(1 + \rho^2/4a^2),$$

and further assuming $\rho^2/4a^2 \ll 1$, then we obtain

$$I(\alpha, \theta, \phi) = 2 \ln (8a/\rho).$$

The assumption of the thin wire leads to

$$G(\alpha, \theta, \phi) = (\cos^2 \theta + \alpha^2 \sin^2 \theta) \sin^2 \phi + \cos^2 \phi$$

Accordingly, the capacitance per unit length of the loop is given in simple form by

$$C = \pi \varepsilon_0 \sqrt{K'K/\ln (8a/\rho)},$$

where

$$K = K_0 [(\cos^2 \theta + \alpha^2 \sin^2 \theta) \sin^2 \phi + \cos^2 \phi].$$

It is noted here that the capacitance of (11) agrees with the electrostatic capacity per unit length of an infinite parallel wire line [13] placed on the $x$-$y$ plane and parallel to the $y$-axis in coordinates as shown in Fig. 1. By using (7) and (11), the characteristic impedance $Z_0$ and propagation constant $k$ are compactly expressed by

$$Z_0 = (1/\pi) \sqrt{\mu_0/\varepsilon_0 K'K} \ln (8a/\rho),$$

$$k = k_0 [K'K]^{1/4}, \quad k = \omega \sqrt{\varepsilon_0 \mu_0}.$$}

Consequently the input impedance of the loop antenna in the zeroth order can be obtained from that of a short-circuited transmission line with length $\pi a$ as follows:

$$Z = jZ_0 \tan \pi a, \quad a/\rho \gg 1, \quad \text{Im} \ (k) \leq 0.$$
TABLE I

<table>
<thead>
<tr>
<th>Magnetic Field</th>
<th>Propagation Constant</th>
</tr>
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<tbody>
<tr>
<td>$K' &gt; 0$</td>
<td>$i): \frac{k}{k_0} = (K'K)^{1/4}$</td>
</tr>
<tr>
<td>$K &lt; 0$</td>
<td>$ii): \frac{k}{k_0} = \frac{(-K'K)^{1/4}}{\sqrt{2}} (1 - j)$</td>
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<tr>
<td></td>
<td>$iii): \frac{k}{k_0} = \frac{(-K'K)^{1/4}}{\sqrt{2}} (1 - j)$</td>
</tr>
<tr>
<td></td>
<td>$iv): \frac{k}{k_0} = (-K'K)^{1/4}$</td>
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</table>

magnetic field. The propagation constant obtained also agrees with that obtained for an infinite thin conductor in a magnetoplasma [14]. The propagation constant (16) is shown in Table I for the collisionless case. It is found from Table I that there exist propagating waves in region i), damping waves in region iv), and complex waves in regions ii) and iii). It is also found that even if $v_t = 0$, the input impedance in the regions ii) and iii) has resistance arising from the complex wave.

Fig. 2 shows the propagation constant in the high frequency region calculated from (16) for the cases of $\theta = 0^\circ$ and $90^\circ$ of $\phi = 90^\circ$. In Fig. 2 $f_p$, $f_c$, $f_{uhr}$ and $\nu$ are electron plasma frequency, electron cyclotron frequency, upper hybrid resonance frequency, and electron collision frequency, respectively. The curves of $k_a = 1/2$, $3/2$ for the circular loop with diameter 20 cm are also shown in the same figure by dot-lines. For the case of $\theta = 0^\circ$ there exist complex waves for $f < f_p < f_c$ and $f_c < f < f_{uhr}$. The waves in the regions of $f_p < f < f_c$ and $f_{uhr} < f$ are respectively the lower and upper propagating modes. On the other hand, when $\theta = \phi = 90^\circ$, the lower propagating mode is shown to exist in the region of $f < f_c$, and the waves in the region of $f_c < f < f_{uhr}$ are converted into the damping mode. In the region of the propagating mode, points of intersection of two curves $\Re(k/k_0)$ and $ka = 1/2$ indicate the half-wavelength resonances of the input impedance. The corresponding resonance frequencies are denoted by $f_{1/2}^{(n)}$ and $f_{u1/2}^{(n)}$ in the figure. Here, $f_{1/2}^{(n)}(n = 1, 2, 3, \cdots)$ is a upper half-wavelength resonance frequency due to a half-wavelength resonance, which is usually obtained in free space, while $f_{u1/2}^{(n)}$ is a lower half-wavelength resonance frequency which is newly predicted impedance resonance of the loop antenna in the lower frequency region. This resonance occurs due to the existence of the lower propagating wave in a magnetoplasma. Generally, for an arbitrary angle $\gamma$ in Fig. 1 $f_{1/2}^{(n)}$ exist in the frequency region of $f_{ir} < f < f_{cr}, f_{ir} < f_{cr} < \min(f_p, f_c)$, where $f_{ir}$ is lower resonance cone frequency determined by $K_0 \sin^2 \gamma + K' \cos^2 \gamma = 0$ in regard to (16).

The input impedance characteristics in the high frequency region are discussed in the following section. Fig. 3 shows the input impedance calculated from (15) for $H^+$ ions in the frequency region near the lower hybrid and ion cyclotron frequencies, where $f_{pe} = 25$ GHz, $f_{ce} = 50$ GHz, $v_t = 0$. From Figs. 3(a) and 3(b) the resonance peak of the impedance at the lower hybrid resonance frequency $f_{hr}$ is found to always exist for arbitrarily oriented antennas of finite size. In the case of $\theta \neq 90^\circ$ resistance $R(R = X)$ due to the complex mode in the frequency region $f < f_{ci}, f_{hr} < f < f_{ir}$ is also shown to exist. Fig. 3(b) shows the impedance for the frequency near

Fig. 2. Normalized propagation constants of the transmission line constituting the loop. The upper half- and the lower half-planes represent the real and imaginary parts of the propagation constants, respectively. Curves for $k_a = 1/2$, $3/2$ are for the circular loop with diameter 20 cm.

Fig. 3. Calculated input impedances of the antenna with the inclination $\theta$ varied in the lower frequency region. $2a = 30$ cm, $2p = 3$ mm, $\phi = 90^\circ$, $v_t = 0$. At the lower hybrid resonance frequency $f_{hr}$ is found to always exist for arbitrarily oriented antennas of finite size. In the case of $\theta \neq 90^\circ$ resistance $R(R = X)$ due to the complex mode in the frequency region $f < f_{ci}, f_{hr} < f < f_{ir}$ is also shown to exist. Fig. 3(b) shows the impedance for the frequency near
$f_{cl}$. The resistance of the loop in the frequency region of $f < f_{cl}$ and $f_{thr} < f < f_{ne}$ has been obtained already by Wang and Bell [8], and predicted by using the second-order quasistatic theory whenever the loop is small and one of the characteristic electromagnetic mode of the plasma has an open refractive index surface. It is also interesting to note that the resistance near $f_{cl}$ is a few ohms and is close to the radiation resistance of a loop antenna used for ICRF heating [15]. When $\gamma = 0$, it is found from Fig. 3(a) that another half-wavelength resonance appears even in the lower frequency region near and below ion cyclotron frequency if the collision loss is neglected.

The input impedance of a small loop antenna is usually given by $Z = j Z_0 \frac{\kappa a}{\kappa a} = j 120 \pi \kappa a \ln(8a/\rho)$, and is plasma-independent [6], [12]. It should be noted, however, that the input impedance for $\gamma = 0$ is not always plasma-independent, and that the new impedance resonance of a lower half-wavelength resonance exists for $f_{ce} \ll f_{pe}$.

III. EXPERIMENTS AND DISCUSSION

The experiments were carried out in the space chamber of the Research Institute of Electrical Communication, Tohoku University. The chamber is 2.5 m in diameter and 4 m in length. The experimental setup and block diagram of measurements are shown schematically in Fig. 4. The experimental setup is the same as used in the measurements of the impedance of wire antennas [16] except for the antennas and system for measuring the impedance. The shielded circular loop antenna is used as the experimental loop antenna.

In order to produce a large and uniform plasma, an RF discharge plasma, whose frequency is 9.1 MHz and maximum power of 500 W, was used with Ar as a discharging gas at a gas pressure about $5 \times 10^{-4}$ torr. Typical plasma parameters are electron density $1.5 \times 10^8$ cm$^{-3}$ and electron temperature $T_e = 6 \times 10^4 K$ [16]. Two sets of magnetic coil are placed to impress the static magnetic field inside the chamber. To suppress the reflected waves from the inner walls of the chamber and the coils, a wave absorbing wall is also placed around the antenna. This absorbing wall is composed of three layers of glass tubes packed with foamed poly styrene particles impregnated with graphite [16].

In Figs. 5 and 6, the input impedances measured in a magnetoplasma when the diameter of the shielded loop is 20 cm are indicated by the open circles. Figs. 5 and 6 are for the cases of the impressed magnetic field normal and parallel to the plane of loop, respectively. The solid and broken curves in these figures indicate the impedance calculated from (15) for $\nu_{op} = 0.05$ and 0.15, respectively. The electron cyclotron frequency $f_{ce}$ used in these calculations is determined from the current of the magnetic coils. The electron plasma frequency $f_p$ and the collision frequency $\nu$ are determined as the best fitting parameter for the measured values in the vicinity of the cyclotron frequency. Thus, the electron plasma frequency was determined as about 150 MHz, although $f_p$ obtained by a double probe measurement was $70 \sim 80$ MHz which is almost half of that determined by the best fitting technique. It is usual that the plasma frequencies determined by different methods vary as much as by factor 2 or 3.

Both Fig. 5 and Fig. 6 are the results measured for $f_p < f_{ce}$.

We found from Figs. 5 and 6 that input impedance does not behave apparently itself like an antiresonance at $f_{thr}$. It should be noted from Figs. 5 and 6 that there are not impedance antiresonances at $f_{thr}$ as to the case of loop antennas in contrast to the dipole antennas, while the impedance resonance at $f_{ce}$ is observed for the loop antenna as the dipole antenna. The halfwavelength resonances for $f_{thr} < f$ are the same as those obtained in free space, and are indicated in the figures as $f_{(1/2)w}$. It is clearly shown in Figs. 5 and 6 that new impedance resonances $f_{(1/2)w}$ for the loop of finite size in the region of lower propagating mode for $f < f_{ce}$ are observed experimentally. These results confirms the resonance phenomena predicted theoretically in Section II, which have not been observed so far in any other experiments on the loop antenna.

Comparing the measured results of Fig. 5 with those of Fig. 6, it is found that measured peak of new resonance of $f_{(1/2)w}$ below $f_{ce}$ for $\theta = 90^\circ$ appears clearly and moves toward a lower frequency than the case for $\theta = 0^\circ$, and that the zero-reactance point of $f_{(1/2)w}$ above $f_{thr}$ for $\theta = 90^\circ$ moves toward a higher frequency than the case for $\theta = 0^\circ$. These experimental results confirms the calculated ones as shown in Figs. 5 and 6 or Fig. 2.

It is found from Figs. 5 and 6 that the measured values agree fairly well with the calculated ones for the both cases of the static magnetic field normal and parallel to the plane of the loop. The discrepancy between measured results and calculated ones in the frequency regions of one wavelength-resonance frequency greater than $f_{thr}$ results from the very approximate theory neglecting the radiation resistance of the antenna, which is particularly important in the higher frequency.

IV. CONCLUSION

An approximate but compact and practically useful formula for the input impedance of a large circular loop antenna with arbitrary orientation in a cold magnetoplasma has been obtained by using the transmission line theory. As a result of
Fig. 5. Experimental and theoretical input impedances of the antenna for the case of the static magnetic field normal to the plane of loop ($\theta = 0^\circ$). $2a = 20$ cm, $2\rho = 3.58$ mm.

The numerical calculation, the new impedance resonances of loop antennas of finite size in a magnetoplasma in the frequency region below and near the electron cyclotron frequency are predicted. The input impedance characteristics in the lower frequency region near the lower hybrid resonance and the ion cyclotron frequency are also obtained.

The input impedances of a large circular loop antenna in a magnetoplasma have been measured in the high frequency region for both cases of the static magnetic field normal and parallel to the plane of the loop. The newly predicted impedance resonances for the antenna of finite size have been observed experimentally. It has been also shown that the measured impedances agree fairly well with the calculated ones.

APPENDIX

The integral $I(\alpha, \theta, \phi)$ of (2) becomes singular (collisionless plasma) by nature at the characteristic frequencies of the lower and upper resonance cone [12]. When $\theta = 0^\circ$, $R(\eta)$ of (4) is expressed as

$$ R(\eta) = 1 - \sin^2 \eta / (1 + \rho^2 \alpha^2 / 4a^2). $$

(17)

We assume $\rho^2 \alpha^2 \ll 4a^2$ or $\rho \ll a$ and $\alpha \neq 0$ in (17), then, we obtain a simple equation for (2) as follows:

$$ I(\alpha, \theta, \phi) = 2[\ln (8a/\rho) - \ln \alpha] $$

$$ = 2 \ln (8a/\rho). $$

(18)
The preceding indicates only the effect of the shape of the loop of finite size in the free space. It can be also obtained if we put \( \alpha = 1 \) and \( \rho \ll a \). As a result, it is found that the approximation of \( \alpha = 1 \), and \( \rho \ll a \) does not affect the integral of (2) for \( \theta = 0^\circ \), and that the integral \( I(\alpha, \theta, \phi) \) for \( \theta = 0^\circ \) can be replaced by the corresponding value for the nonmagnetized plasma. Therefore, in the integral \( I(\alpha, \theta, \phi) \) of (2), we extend the assumption of \( \alpha = 1 \) and \( \rho \ll a \) to the cases of any \( \theta \) for simplicity.

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REFERENCES


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