vals to speed up the adaptation process even if only approximate directions of interferences are known.

IV. CONCLUSION

The method of alternating orthogonal projections proposed in this paper is shown to be a very simple technique for synthesis of "null-steering" patterns for arbitrary array geometries. The method is also seen to have applications in adaptive over a ground plane covered with a magnetoplasma with any arbitrary direction of the static magnetic field is presented. The analysis is restricted obtained. For other directions of the static magnetic field, the reflection field with respect to the ground plane perfect mirror reflections can be obtained. These modifications can be significant under hyperbolic plasma conditions. A short and a long monopole were considered. From observations, the resonance cone and the resonances corresponding to the antenna length have been observed.

REFERENCES


Impedance of a Monopole Antenna over a Ground Plane and Immersed in a Magnetoplasma

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Abstract—A treatment of the input impedance of a monopole antenna over a ground plane covered with a magnetoplasma with any arbitrary direction of the static magnetic field is presented. The analysis is restricted to a cold plasma with uniaxial and quasi-static approximations. It has been found that for parallel and perpendicular directions of the static magnetic field with respect to the ground plane perfect mirror reflections can be obtained. For other directions of the static magnetic field, the reflection is birefringent so that the monopole impedance becomes modified over a dipole impedance. These modifications can be significant under hyperbolic plasma conditions. A short and a long monopole were considered. From laboratory measurements of a long monopole impedance, the resonance cone and the resonances corresponding to the antenna length have been observed.

I. INTRODUCTION

The importance of the knowledge of antenna impedances in magnetoplasmas resulted in a large number of papers. After the short antenna theory of Balmain [1] through the interpretation of the quasistatic theory which predicts radiations and propagations and provides a very convenient method of impedance formulation of an antenna of arbitrary geometry, the current distribution and the input impedance of a linear antenna have been studied by Mushiake [2]; Hurd [3]; Chen and Seshadri [4]; Seshadri [5]; Galejs [6], [7]; Lee and Lo [8]; Ito and Mushiake [9]; Lafon and Weil [10]; Ishizone, Adachi, and Mushiake [11]; Lee [12]; Lu and Mei [13]; and Adachi, Ishizone, and Mushiake [14]. Through these papers a literature on impedance formulation in a magnetoplasma has been established. On the other hand, input impedance of a short antenna has been measured by Balmain [1] and Bhat and Rao [15]. The impedance of a cylindrical monopole whose length is comparable to a free-space wavelength has been measured by Ito and Mushiake [9]. In these experiments the monopole antennas have been placed parallel to the static magnetic field. Nakatani and Kuehl [16] have measured the input impedance of a short dipole and compared it with the theoretical results of [17] and those of Balmain [1]. Impedance measurements in the ionosphere have been performed by Ejiri, Oya, and Obayashi [18] and Meyer and Vernet [19]. However, these papers treated short antennas or antennas oriented parallel or perpendicular to the geomagnetic field. Recently Sawaya, Ishizone, and Mushiake [20] measured an electrically long monopole antenna with an arbitrary direction of the static magnetic field and compared the results with a dipole imped-
ance calculated by using a uniaxial approximation with a finite magnetic field and a sinusoidal current distribution. However, the difference between a monopole impedance and a dipole impedance for arbitrary directions of the static magnetic field has not been clarified. It seems that a monopole antenna over a ground plane requires a separate impedance analysis because its ground plane cannot always be expected to give a perfect mirror reflection. Moreover, the antennas on the vehicle surfaces of rockets and satellites are usually monopole-type antennas, and each wing of the antenna structure can be treated as a monopole over the interfaces of density irregularities where scattering of radio waves can occur. Under these considerations we decided to study the impedance of a monopole antenna for any arbitrary direction of the static magnetic field. In this paper we shall consider a case where an arbitrary direction of the static magnetic field will create a lack of field symmetry about the monopole axis so that a birefringent reflection can occur at the surface of the ground plane. For simplicity, the analysis is rather straightforward and was performed in light of the existing theories of impedance formulations. In order to interpret the resulting modifications we could not avoid comparison with a dipole impedance available under similar assumptions and approximations. Since the two models can have different field systems a direct comparison has been avoided.

II. CALCULATIONS OF THE MONOPOLE IMPEDANCE

Fig. 1 indicates the geometry of the monopole antenna. The cylindrical monopole axis is aligned with the z axis of a rectangular coordinate system, and the ground plane coincides with the xy plane. The monopole is surrounded by a cold magnetoplasma with the static magnetic field \( \mathbf{H}_0 \) directed at an arbitrary direction \( \theta \) with respect to the monopole axis. For a straightforward impedance calculation we look for the field solution of an elementary current source \( J = \delta(x) \delta(y) \delta(z - L) \), where \( \delta \) is Dirac's delta function. The field at any point in the medium can be classified into the primary field corresponding to the waves traveling directly from the source into the medium and the secondary field resulting from the reflections at the ground plane. Assuming that the sheath is removed by a dc bias the field conditions at the surface of the ground plane are such that the tangential components of the total electric field should vanish. An exact solution of the problem is rigorous for both fields and can be a subject of optics. However for impedance calculations, assumptions and approximations have to be adopted. For simplicity of the analysis we assume that the anisotropic plasma is characterized by a uniaxial dielectric tensor as has been used by Ito and Mushiake [9] and Lee [12]. With reference to the principal axes of the medium,

\[
\mathbf{\hat{e}} = \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_3' & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix}
\]

where

\[
\epsilon_1 = 1 - \frac{L}{1 - M^2}, \quad \epsilon_3' = 1 - L, \quad L = \frac{\omega p^2}{\omega (\omega + jv)}, \quad M = -\frac{\omega C}{\omega - jv}.
\]

For a straightforward introduction of the tensor (1) into the wave equation does not directly provide the eigenfunctions satisfying the boundary conditions at \( z = 0 \), because the principal axes of the medium rotate with the arbitrary directions of the static magnetic field. Hence for convenience we transform it through the fixed coordinate system of the monopole such that the static magnetic field rotates along the yz plane,

\[
\mathbf{\hat{e}} = \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & \epsilon_6 \\ 0 & \epsilon_6 & \epsilon_3 \end{bmatrix}.
\]

The resulting transformations are

\[
\epsilon_2 = \epsilon_1 \cos^2 \theta + \epsilon_3' \sin^2 \theta, \quad \epsilon_3 = \epsilon_1 \sin^2 \theta + \epsilon_3' \cos^2 \theta, \quad \epsilon_6 = (\epsilon_3' - \epsilon_1) \sin \theta \cos \theta.
\]

Using (2) and by a Fourier analysis, the field solutions can be obtained in integral forms which will involve one singularity at infinity. For impedance calculations this should be removed. Hence for convenience we write the fields with their scalar and vector potentials:

\[
E^p = -\nabla \phi^p - j \omega A^p, \quad E^s = -\nabla \phi^s - j \omega A^s.
\]

In order to satisfy the boundary conditions at the surface of the ground plane it is sufficient that

\[
\phi^p + \phi^s = 0, \quad z = 0
\]
where \( K_{31}' \) and \( K_{32}' \) are the roots of the characteristic equation and correspond to the O-mode and the X-mode, respectively,

\[
\begin{align*}
K_{31} & = \pm \sqrt{e_1 K_0^2 - (K_2^2 + K_2^2)} , \\
K_{32} & = -c K_2 \pm \sqrt{d K_0^2 - (a K_2^2 + b K_2^2)} ,
\end{align*}
\]

From (10), \( K_{32}^+ \neq - K_{32}^- \) indicates that the positively traveling and negatively traveling waves cannot have similar behavior. We have chosen that the negative waves are proceeding towards the ground plane and the positive waves are traveling away from the ground plane. In both cases the waves should be attenuating so that \( \text{Im} \left( K_{31,2}^z \right) \leq 0, z > z_0 \) and \( \text{Im} \left( K_{31,2} \right) > 0, z < z_0 \). The functions \( X, Y, \) and \( Z \) appear from the cofactors of the wave matrix, and \( D_1(K_{31,2}^z) \) and \( D_2(K_{31,2}^z) \) are the characteristic functions corresponding to the O-mode and the X-mode, respectively. Denoting \( K_3 = K_{31}^z, K_{32}^z, \)

\[
\begin{align*}
X(K_3) & = (e_1 - e_3) K_2 \cos \theta - K_2 \sin \theta \sin \theta , \\
Y(K_3) & = (e_1 - e_3) (K_2^2 + K_2^2 - e_1 K_0^2) \cos \theta \sin \theta , \\
Z(K_3) & = e_2 (K_2^2 + K_2^2 - e_1 K_0^2) + e_1 K_2^2 , \\
D_1(K_3) & = \sqrt{e_1 K_0^2 - (K_2^2 + K_2^2)} (K_3^\pm - K_{32}^+) (K_3^\pm - K_{32}^-) , \\
D_2(K_3) & = \sqrt{d K_0^2 - (a K_2^2 + b K_2^2)} (K_3^\pm - K_{32}^+) (K_3^\pm - K_{32}^-) .
\end{align*}
\]

\( \phi \) and \( A \) can be defined by similar planewave analysis introducing weighting functions:

\[
\begin{align*}
\phi^P & = \frac{1}{ \omega_0 e_3 } \int_{-\infty}^{\infty} dK_1 dK_2 e^{-jK_1 x - jK_2 y} \begin{bmatrix} X(K_{31}^+) & X(Y(K_3)^+) & Z(K_{31}^+) & Z(K_{32}^+) \end{bmatrix} , \\
A^P & = \frac{1}{4 \pi^2} \int_{-\infty}^{\infty} dK_1 dK_2 e^{-jK_1 x - jK_2 y} \begin{bmatrix} P(K_{31}^+) & P(K_{32}^+) \end{bmatrix} \\
& \begin{bmatrix} A(K_{31}^+) & A(K_{32}^+) \\ B(K_{31}^+) & B(K_{32}^+) \end{bmatrix} \\
& \begin{bmatrix} X(K_{31}^+) & X(K_{32}^+) \\ Y(K_{31}^+) & Y(K_{32}^+) \\ Z(K_{31}^+) & Z(K_{32}^+) \end{bmatrix} \\
& \begin{bmatrix} P(K_{31}^+) & P(K_{32}^+) \end{bmatrix} \\
& \begin{bmatrix} A(K_{31}^+) & A(K_{32}^+) \\ B(K_{31}^+) & B(K_{32}^+) \end{bmatrix} \\
& \begin{bmatrix} X(K_{31}^+) & X(K_{32}^+) \\ Y(K_{31}^+) & Y(K_{32}^+) \\ Z(K_{31}^+) & Z(K_{32}^+) \end{bmatrix} ,
\end{align*}
\]

where \( P(K_{31}), P(K_{32}) \), \( A(K_{31}), A(K_{32}) \), and \( B(K_{31}), B(K_{32}) \) are weighting functions, \( P \) can be determined from (5) and \( A \) and \( B \) from (6),

\[
\begin{align*}
P(K_{31}, K_{32}) & = \frac{1}{2 \omega_0 e_3} \int_{-\infty}^{\infty} e^{-jK_0^2 - (a K_2^2 + b K_2^2)} , \\
A(K_{31}, K_{32}) & = \frac{1}{2 \omega_0 e_3} \int_{-\infty}^{\infty} e^{-jK_0^2 - (a K_2^2 + b K_2^2)} , \\
B(K_{31}, K_{32}) & = \frac{1}{(1/D_1(K_{31}^+) e^{-jK_{31}^+})} , \\
\end{align*}
\]

With variable transformations

\[
\sqrt{\alpha K_1} = K \cos \phi , \quad \sqrt{\beta K_2} = K \sin \phi , \quad \frac{x}{\sqrt{a}} = R \cos \phi , \quad \frac{y - c(z - z_0)}{\sqrt{b}} = R \sin \phi .
\]

Equations (7) and (11) can be converted into Sommerfeld type of integrals that can be obtained as

\[
\begin{align*}
\phi^P & = \frac{1}{j \pi \omega_0 e_3} \int_{-\infty}^{\infty} e^{jK_0} e^{jK_{31}^+ R_1} , \\
\phi^S & = \frac{1}{j \pi \omega_0 e_3} \int_{-\infty}^{\infty} e^{jK_0} e^{jK_{32}^- R_2} ,
\end{align*}
\]

where

\[
\begin{align*}
R_1 & = \sqrt{x^2 + \frac{a}{b} (y - c(z - z_0))^2 + a(z - z_0)^2} , \\
R_2 & = \sqrt{x^2 + \frac{a}{b} (y - c(z - z_0))^2 + a(z + z_0)^2} .
\end{align*}
\]
Again the introduction of $A$ and $B$ from (14) into (12) indicates that $A^S$ vanishes at infinity with respect to $K_1$ and $K_2$, and the corresponding reflected field consists of four waves with phase variations as:

$$e^{-jK_31^+z+jK_31^-z_0}, e^{-jK_32^+z+jK_32^-z_0}, e^{-jK_33^+z+jK_33^-z_0}, e^{-jK_34^+z+jK_34^-z_0}.$$ 

This is expected because there should generally be a birefringence or a double fringe corresponding to each incident mode in the reflected field in an anisotropic medium. These refer to the $OO, OX, XO,$ and $XX$ rays as could be traced by a ray optical description. However, the scalar potential contains a single mode, the $X$-mode only, because it satisfies the boundary condition with a single arbitrary constant. The field symmetry about the monopole axis can be checked. For $\theta = 0^\circ$ and $90^\circ$, $c$ vanishes so that $\phi^P$ and $\phi^S$ become symmetrical about the $z$ axis. For similar reasons $K_3^2 = -K_3^2$ so that $A^P$ becomes symmetrical. Again $N_{12}$ and $N_{21}$ vanish so that the rays $OX$ and $OX$ will disappear giving a field symmetry and a perfect mirror reflection. These field symmetries are in agreement with Rao and Wu [21].

One advantage of calculating the vector potential is that although the impedance can be calculated from the quasi-static field, it can be explained by looking at the vector potential because in the low frequency limit the contribution of the vector potential to the power flow will be equal to that of the quasi-static field through its "rotational induced current" as has been identified by Balmain [1]. The quasi-static solution follows readily from the solution stated above through the quasi-static limit:

$$\phi^S = \frac{1}{j4\pi \omega_0 \sqrt{\varepsilon_1 \varepsilon_3'}} \hat{e}_z \left( \frac{1}{R_1} - \frac{1}{R_2} \right). \quad (19)$$

However, it has no restriction on the gyrotropic elements of the dielectric tensor because it can also be obtained as a solution of Poisson's equation in a magnetoplasma satisfying the boundary condition at $z = 0$. The first term corresponds to the primary field and is identical to the near-field solution obtained by Mittra and Deschamps [22]. It can be observed that the appearance of $z = z_0$ in $R_2$ displaces the mirror-image location $(0, -z_0)$ by a distance $2cz_0$ measured parallel to the ground plane. A similar phenomenon for a magnetic line source has been observed by Felsen [23]. The corresponding reflection mechanism can be schematized in Fig. 2 for the hyperbolic regions with $\cos^2 \theta + (\varepsilon_1/\varepsilon_3') \sin^2 \theta > 0$. The propagation modes are restricted to ray angles satisfying the condition $\phi < \phi_c = \tan^{-1} |\varepsilon_1/\varepsilon_3'|^{1/2}$ where $\phi$ is measured from the positive $H_0$ axis. Consequently, the direct ray $S_1$ from the source illuminates only the region $P$, while propagating rays $S_2$ from the image source, i.e., the reflected rays from the ground plane, illuminate both regions $P$ and $Q$. The shaded area indicates the shadow zone in which the fields decay. It can be noted that the presence of the ground plane enlarges the illuminated region and both the incident and reflected rays lie on the same side of the normal to the ground plane—a phenomenon that cannot be encountered in isotropic media.

The input impedance of a short monopole with a triangular current distribution can be obtained in an approximate closed form:

$$Z_m = \frac{60 \sqrt{\varepsilon_1/\varepsilon_3'}}{jK_0 \varepsilon_1 A_1 \left( \ln \frac{1}{\rho} - \ln \frac{A_1 + \sqrt{\varepsilon_1/\varepsilon_3'}}{A_1^2} \right)} \left( -1 + \frac{2\sqrt{\varepsilon_1/\varepsilon_3'}}{A_1 A_2 + \sqrt{\varepsilon_1/\varepsilon_3'}} \right), \quad (20)$$

where

$$A_1 = \frac{\sqrt{\sin^2 \theta + \varepsilon_1 \cos^2 \theta}}{\varepsilon_3},$$

$$A_2 = \frac{\sqrt{\cos^2 \theta + \varepsilon_1 \sin^2 \theta}}{\varepsilon_3}. $$

Positive real parts of the square roots should be taken. As a monopole and a dipole can have different field conditions for arbitrary directions of the static magnetic field as long as the transverse plane passing through their driving points is concerned, the two models should be treated as different and a direct comparison may be avoided. With a common factor the first three terms in (20) stand for the half of a dipole impedance obtained by Balmain [1]. The fourth term accounts approximately for the effect of the double fringe by reflection at the ground plane, and it vanishes for $\theta = 0^\circ$ and $90^\circ$ giving a perfect mirror reflection so that the first three terms will determine the monopole impedance for these symmetrical cases. For other directions of the static magnetic field the numerical estimation of the double fringent term can be taken from Figs. 2-4. The solid lines represent a monopole impedance, and the dotted lines indicate the half of a dipole impedance for similar parameters. At frequencies above the upper hybrid resonance $f_h$ where the medium is nearly isotropic the two curves have almost equal values and therefore have been shown with solid lines. Up to the lower resonance cone $f_{res}$ there is a good agreement between the monopole and dipole impedances. Although the models are different they take waves from the same dispersion equation.
Fig. 3. Monopole and dipole impedance by quasi-static approximation as a function of frequency. (a) $\theta = 30^\circ$, $f_p = f_c$. (b) $\theta = 60^\circ$, $f_p = f_c$.

Fig. 4. Monopole and dipole impedance by quasi-static approximation as a function of frequency. (a) $\theta = 30^\circ$ and $f_c > f_p$. (b) $\theta = 60^\circ$ and $f_c > f_p$. 

so that there is no shift of the resonances pertaining to the properties of the medium. The major differences in impedance behavior can be noted around and below $f_{rca}$. As in Fig. 3(a) the monopole resistance increases at and below $f_{rca}$ and then has a sharply decreasing tendency to meet with the reactance below. In this region the monopole reactance also differs considerably from the dipole reactance and becomes inductive. These tendencies are reverse to the impedance behavior of the dipole and remain unchanged for zero collision frequency.

The effect of increasing $\theta$ can be observed in Fig. 3(b). Here also the resistance is modified inside and below the resonance cone, and the sharply decreasing tendency of the monopole resistance can be noted, the reactance being inductive. Figs. 4 and 5 have been drawn for increased strength of the static magnetic field. It can be noted that the impedance behavior of the monopole does not change except that the zero-reactance point below $f_{rca}$ is shifted slightly to the right. The decreasing tendency of the monopole resistance below the lower resonance cone arises from the fact that $\varepsilon_3'$ becomes very large in this region and $|\varepsilon_1/\varepsilon_3'| \ll 1$ so that the contribution of $\ln \sqrt{\varepsilon_1/\varepsilon_3}$ in the fourth term of (20) can considerably reduce the contribution from the other terms to yield a minimum resistance which for $\theta = 0^\circ$ and $90^\circ$ can occur at a frequency

$$f \approx 0.68 \frac{f_{rca} \rho}{\sqrt{f_{rca}^2 + f_c^2}} \frac{\rho}{l} \cos \theta.$$  

Hence with increased $l/\rho$ it becomes shifted to a very low frequency. One physical explanation of this can be given as follows. Referring to Fig. 2, at very low frequency, the opening angle $\phi_c = \tan^{-1} \sqrt{|\varepsilon_1/\varepsilon_3'|}$ of the characteristic cone becomes extremely small so that the incident ray $S_1$ and the reflected ray $S_2$, both being on the same side of the normal, can cancel out each other. Hence the field will be determined by the direct rays which decay because they become engulfed by the shadow region of the source image. With such reduction of the field a minimum resistance from image displacement can occur at very low frequencies. Another anomaly can be noted when $A_2 = 0$, i.e., when $\tan^2 (90^\circ - \theta) = -\varepsilon_1/\varepsilon_3'$. At the corresponding frequencies a characteristic cone emanating from the driving point of the monopole coincides with the ground plane so that an oblique resonance at the ground plane can be expected. In Figs. 3-5, the corresponding frequencies have been shown as $f_{rca}$ where the expected anomaly can be noted. It lies around the monopole oblique resonance $f_{rca}$, and for $\theta = 45^\circ$ it coincides with $f_{rca}$. Similar resonances can be found around the upper resonance cone which have not been shown.

In order for the antenna to radiate in the elliptic region, i.e., between $f_p$ and $f_c$, and also above the upper hybrid resonance, the quasi-static solution seems inadequate for this purpose. Also a triangular current distribution cannot predict the probable antenna-length effects of an electrically long antenna. The primary solution by the uniaxial approximation can be obtained in closed form which will be identical to that obtained by Clemmow [24], and the $z$-component of the field can be given as

$$E_z = -\frac{j \omega \mu_0}{4\pi \sqrt{\varepsilon_1 \varepsilon_3}} \left( \frac{1}{K_0^2 \varepsilon_2} \frac{z^2}{\sqrt{z^2 + \varepsilon_2}} \right) e^{-j K_0 \sqrt{\varepsilon_3} R_1}$$

$$+ \frac{\omega \mu_0 \sin^2 \theta \hat{c} x}{4\pi K_0 \sqrt{\varepsilon_1}} \frac{x}{R^2} (e^{-j K_0 \sqrt{\varepsilon_1} r_1} - e^{-j K_0 \sqrt{\varepsilon_3} R_1}),$$  

$$f_{p} = 3.0 \text{ MHz}$$

$H_0 = \infty$

$\omega = 0.2 \omega_p$

$\theta = 30^\circ$

$l = 50 \text{ cm}$

$\rho = 1 \text{ cm}$

$\theta = 30^\circ$

Fig. 5. (a) Monopole and dipole resistance by quasi-static approximation as a function of frequency for an infinite magnetic field. (b) Monopole and dipole reactance by quasi-static approximation as a function of frequency for an infinite magnetic field.
where
\[ r_1 = \sqrt{x^2 + y^2 + (z - z_0)^2}, \]
\[ R = \sqrt{x^2 + \{y \cos \theta - (z - z_0) \sin \theta\}^2}. \]

Regarding the secondary field there is no pole singularity in the integral expression of \( \mathbf{A}^2 \), and it is suitable for a numerical impedance analysis. We assume a sinusoidal current distribution with the propagation constant obtained by Hurd [3] and Ishizone, Adachi, and Mushiake [11] for a linear antenna
\[ \beta = \kappa_0 [\epsilon_1 (\epsilon_1 \cos^2 \theta + \epsilon_3 \sin^2 \theta)]^{1/4}, \tag{22} \]
where \( \beta \) is the propagation constant that corresponds to that obtained by Mushiake [2] for \( \theta = 0^\circ \) and to that obtained by Galejs [7] for \( \theta = 90^\circ \). The fourth root of \( \beta \) has been taken such that \( \text{Im}(\beta^2) < 0 \) and \( \text{Im}(\beta) < 0 \). Then with the electromotive force (EMF) method a numerical impedance analysis has been performed. A typical impedance plot for \( \theta = 45^\circ \) and with a finite magnetic field is shown in Fig. 6 with solid lines. The dotted lines indicate a dipole impedance for similar parameters calculated by Sawaya, Ishizone, and Mushiake [20] by the EMF method using a current distribution as stated above with a field solution identical to that obtained by Clemmow [24]. The effect of increasing the antenna \( \rho / \rho \) can be observed from comparisons of Figs. 3-5 with Fig. 6. The plasma parameters used in Fig. 6 are of similar orders if normalized by the plasma frequency \( f_p \). The decreasing tendency of the monopole resistance and the inductive reactance below the lower resonance cone can be noted. The maxima and minima of the impedance at the resonance cone have been smoothened to form a uniform spike. This results from increasing the \( \rho / \rho \) of the antenna. Similar smooth behavior can be obtained by increasing \( \rho / \rho \) for the short antenna described by (20). The impedance spike near \( f_c \) is the half-wavelength resonance (\( \text{Re}(\beta) = \pi \)) of the monopole which fits with the one-wavelength resonance of the dipole. In Fig. 6(b) the zero-reactance point of the monopole near 170 MHz corresponds to the quarter-wavelength resonances that fit with the half-wavelength resonance of the dipole. It can be noted that the difference in field system of the monopole and the dipole creates a negligible difference in their impedance values around these resonances. In Fig. 6(a) the difference of monopole and dipole resistance above the upper resonance cone \( f_{\text{ure}} \) can be noted. The decreasing tendency of the resistance above the upper resonance cone has a similarity with the corresponding decrease below the lower resonance cone. These can also be checked by (20). Thus the major differences between the monopole and dipole impedances can be noted at the extremeties of the hyperbolic regions where \( |\epsilon_1 / \epsilon_3| \) becomes very small.

III. EXPERIMENTAL INVESTIGATIONS

The purpose of this investigation is to compare the measured impedance of a monopole antenna with its theoretical impedance results. The impedance measurements of Sawaya, Ishizone, and Mushiake [20] using a monopole antenna are in agreement with the present calculations indicating that the measured data can be interpreted with a sinusoidal current distribution and a uniaxial approximation with a finite magnetic field. However, in those investigations the measurements

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**Fig. 6.** (a) Monopole and dipole resistance by uniaxial approximation as a function of frequency using a finite magnetic field,
\[ \theta = 45^\circ, \quad f_p = 156 \text{ MHz}, \quad \text{and} \quad f_c = 208 \text{ MHz}. \]

(b) Monopole and dipole reactance by uniaxial approximation as a function of frequency using a finite magnetic field,
\[ \theta = 45^\circ, \quad f_p = 156 \text{ MHz}, \quad \text{and} \quad f_c = 208 \text{ MHz}. \]
at lower frequencies where the effect of an ion sheath is assumed to be strong were not performed. We shall extend those measurements to lower frequencies taking into account the effect of the sheath. The experimental setup is the same as used in [20] except that the impedance measurements between 30 MHz to 110 MHz were taken with an admittance bridge, and those above 110 MHz were taken with a network analyzer. This was because of the ratings for stable operation of the instruments. The measured antenna was a monopole of length 30 cm and radius 1 mm erected on a ground plane of about 1 X 1.2 m. The monopole was oriented at an angle of \( \theta = 45^\circ \) with respect to the axis of the coils of the static magnetic field. A typical plot of the measured results is shown in Fig. 7. The plasma and collision frequencies were determined by curve-fitting around the upper hybrid resonance region. The cyclotron frequency was calculated from the coil currents. With the parameters so determined the theoretical curves were drawn as indicated by the solid lines. The collision frequency obtained by such curve-fitting was compared with its value calculated from the collision cross section. It was found that the observed value of the collision frequency was about three to five times greater than that obtained from the collision cross section. Sawaya et al. [20] also observed such high collision phenomena. However, in the present study since the theoretical peak of the monopole resistance at the upper hybrid resonance is about ten percent below the dipole peak, then by similar curve-fitting the collision frequency was found to be less. The theoretical curves were found to be in good agreement with the measured data except in the reactance below the resonance cone (Fig. 7(b)). This is attributed to an effect of the sheath. Hence we calculated an effect of the sheath by considering two coaxial cylindrical monopoles, the outer monopole being surrounded by the magnetoplasma and the inner monopole by free space. The intermediate space between them was assumed to correspond to the thickness \( s \) of the sheath calculated from Debye length, and the tangential field components along the surfaces of the two monopoles were assumed to vanish. The impedance considering the effect of the sheath is shown by dotted lines in Fig. 7. The dotted curves agree well with the measured reactance at low frequency. However, the resistance does not seem to agree quantitatively below the resonance cone and at the upper hybrid resonance with the dotted lines. A better agreement can be found if the collision frequency is reduced. The oblique resonance near 110 MHz is in agreement with the experimental data. The zero-reactance point near 170 MHz corresponds to the quarter-wavelength resonance where \( \text{Re}(\beta) = \pi/2 \). The half-wavelength resonance near 210 MHz of the measured data is shifted to the right of the theoretical one. An insulated sheath model as suggested by Galejs [6] might have explained this. In Fig. 7(b) the solid lines and the dotted lines were supposed to be identical at higher frequencies. The difference is attributed to an error in using a capacitive sheath model at frequencies where a short antenna limit is not valid. However, at low frequencies it is exact because of the short antenna length. The distortions in the impedance measurements at low frequencies are notable. These are attributed to low frequency noise from inhomogeneities. The appearance of the sheath and the inhomogeneities indicates that a radio frequency (RF) discharge cannot remove the nonuniformity of the plasma.

![Theoretical](image1)
![Theoretical](image2)

**Fig. 7.** (a) Comparison between measured and theoretical input resistance for a monopole antenna in a magnetoplasma. (b) Comparison between measured and theoretical input reactance for a monopole antenna in a magnetoplasma.

### IV. CONCLUDING REMARKS

The inapplicability of the conventional image technique in an anisotropic medium resulted in a separate impedance analysis for a monopole antenna in a magnetoplasma. The analysis is restricted to uniaxial and quasi-static approximations. It was found that for parallel and perpendicular directions of the static magnetic field with respect to the ground plane perfect mirror reflections can be obtained. These are in agreement with the predictions of Felsen [23] and Rao and Wu [21]. For other directions of the static magnetic field a birefringent reflection at the ground plane can occur, and these reflections can be effective under hyperbolic plasma conditions as can be estimated from the quasi-static approxi-
mation. The vector potential of the reflected field has been given in integral form which can be asymptotically evaluated for studying the radiation pattern of a monopole antenna. Impedance of a short and a long monopole antenna has been calculated, and the resulting modifications for arbitrary directions of the static magnetic field have been studied in comparison with a dipole impedance. Towards the extremities of the hyperbolic regions significant differences between a monopole and dipole impedance can occur. In those regions the monopole resistance shows a tendency to decrease and the reactance is inductive. Below the lower resonance cone this behavior is reverse to the impedance behavior of a dipole. The antiresonance occurring when the monopole axis coincides with a characteristic cone has been found to be sensitive to the antenna dimensions, and the presence of a reflecting boundary seems to modify their energy. Additional oblique resonances can occur from the ground plane of the monopole. A laboratory measurement of a long monopole impedance has been performed. From the agreement between the theoretical and experimental results the resonance cone and the resonances corresponding to the antenna length have been observed. The low frequency minimum resistance does not lie in the measured range of frequencies. However, the effects of inhomogeneities can be important limiting factors in this region. The appearance of the ion sheath indicates that an RF discharge cannot remove the nonuniformity of the plasma. The resonances corresponding to the antenna length are effects of current distribution of an electrically long antenna. The uniaxial approximation with a finite magnetic field may be a simple approach for interpreting the measured impedance results for a wider range of frequencies.

ACKNOWLEDGMENT

The authors are thankful to Professor Saburo Adachi of the Department of Electrical Engineering, Tohoku University, for valuable suggestions and encouragement with this research. The authors wish to thank Mr. Hitoshi Ishikawa, Mr. Makio Tsujiya, and Mr. Ikuo Okumura for their assistance in the experiments.

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