Rotationally Reconfigurable Single-element Prism for Enhancing Scanning Flexibility of Risley-prism Antenna System

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Abstract—In this letter, we propose a rotationally reconfigurable single-element prism that is compatible with the typical Risley prism antenna system. It consists of a pair of rotating decentered phase plates operating at 30 GHz. The equivalent prism angle can be continuously changed between 0 and 14 degrees by simply rotating these two phase plates. An all-dielectric low-cost prototype has been designed, fabricated, and measured, which demonstrates the peak realized gain of ~21 dBi can be achieved with a gain variation less than 0.9dB within the reconfigurable angle range of 0 to 14 degree. Besides, the combination of phase distribution of two rotating phase plates can be used as a new mechanism avoiding using first-order paraxial approximation to realize 2D beam scanning.

Index Terms—Mechanical beam scanning, phase gradient plate, reconfigurable prism antenna, Risley prism antenna.

I. INTRODUCTION

C imple and low-cost beam-scanning antennas have received Dimuch research attention in the next generation (beyond 5G/6G) wireless communication applications for flexible coverage[1]. Generally, based on the antenna control system, beam scanning antennas can be divided into two categories, namely electronically scanning antenna (ESA) and mechanically scanning antenna (MSA). Although the steered beams can be quickly obtained, the ESA requires many active elements such as switch and phase shifter to achieve the electronically scanning [2], which makes the ESA complex and expensive, particularly in millimeter wave bands. From this perspective, the MSA provides a relatively inexpensive and low complexity alternative solution in some application. Among the MSA, a popular configuration uses Risley prism antenna [3]-[10] with a pair of rotating linear phase gradient (LPG) plate to realize desired elevation and azimuth angles θ and ϕ within the conical beam scan range, as shown in Fig. 1(a).

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Fig. 1. (a) The typical Risley prism antenna system with a pair of rotating linear phase gradient plate. The white arrow represents the direction of linear phase gradient (b) Schematic diagram of proposed rotationally reconfigurable single-element prism. Different prism angles can be realized by rotating a pair of decentered phase plate.

The maximum apex angle range of conical beam scan range is 2δ , and directly determined by the prism angle (δ). The relationship between prism angle and the phase gradient of phase plate ($G_p = d\varphi_p / dx$) at deigned wavelength (λ) can be written as [11].

$$\delta = \arcsin(\lambda G_p / 2\pi) \tag{1}$$

Although different approaches using phase shift surface [3][4][6] and dielectric plates [9] to achieve Risley prism antenna at different operating frequencies, to the best of our knowledge, little attention is focused on exploring rotationally reconfigurable prism or phase plate for further enhancing scanning flexibility of Risley prism antenna. The mechanism of beam scanning in previous studies is the same, and based on the first-order paraxial approximation [6, eq. (1)-(7)]. Besides, an important prerequisite for designing the LPG plate in these studies is that the prism angle and rotational speed of two corresponding prisms are the same. It greatly limits the scanning flexibility of the Risley prism antenna. In fact, by using prisms with different prism angles at different rotational speed (ω), the interesting scan paths or scan patterns can be produced as shown in Fig.2. The similar conclusion can be found in [12]. On the one hand, the rich set of generated scan patterns could provide a shortest path to achieve the required scanning angle, which enhances the agility of system. On the other hand, according to specific requirements such as the

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Fig. 2. The set of generated scan patterns by using prisms with different prism angles at different rotational speed. (a) M = 1, N = 1. (b) M = 1, N = 2. (c) M = 1, N = 6.(d) M = 0.5, N = 4.(e) M = 1.5, N = 4.(f) M = 5, N = 4. Noted that the ratio of prism angle for two prisms is $M = \delta/\delta'$, and the ratio of rotational speed for two prisms is $N = \omega/\omega'$.

population density on the street, the radiated beam could be simply adjusted to cover specific areas for power saving, which further improves the usability of the system to different wireless application scenarios at a low-cost. Therefore, to realize a more powerful Risley prism antenna system, a single-element prism with rotationally reconfigurable function is particularly important and necessary.

In this letter, a rotationally reconfigurable single-element prism that is compatible with the typical Risley prism antenna system is proposed, which consists of a pair of rotating decentered phase plates at deigned frequency of 30 GHz. The equivalent prism angle can be continuously changed by simply rotating these two decentered phase plates, as shown in Fig.1(b). In other words, different combinations of these two decentered lenses can be regarded as different equivalent prisms. Besides, the combination of phase distribution of two rotating phase plates can be used as a new mechanism avoiding using the first-order paraxial approximation to realize 2D beam scanning, providing a potential mechanism for beam scanning antenna systems.

II. RECONFIGURABLE SINGLE-ELEMENT PRISM DESIGN

A. The Phase Function of Rotating decentered phase plate

As is well known, the beam deflection in a specific direction like a prism can be achieved by the lateral shift of two refractive lenses elements with respect to each other in Galilean afocal lens pair system [13, 14], as shown in Fig.3(a). In this lens pair, diverging lens has negative optical power, while converging lens has positive optical power. The schematic of corresponding phase distribution for these two lenses is shown



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Fig. 3. (a) The beam deflection in a specific direction like a prism using Galilean afocal lens pair system with opposite optical powers. (b) Schematic of phase distribution for the lens pair with opposite optical powers. Left: diverging lens; Right: converging lens. (c) Schematic of the corresponding decentered phase distribution.

respectively in Fig.3(b). However, the characteristics of lateral offset determines that it is not compatible with typical Risley prism antenna using rotation operation. In order to solve this problem, two decentered phase plates can be utilized to change the lateral shift operation into mutual rotation operation. The corresponding decentered phase distribution for the lens pair with opposite optical powers is illustrated in Fig.3(c).

For a general case, the phase plate 1 is rotated around the central axis by an angle of $\theta_r/2$ in a counterclockwise direction, and the phase plate 2 is rotated around the central axis by an angle of $-\theta_r/2$ in a clockwise direction. The general form of phase function of decentered phase plate 1 and 2 in cartesian coordinate system is respectively defined by the following equation.

$$P_{1,\theta_r/2} = -\frac{\pi}{\lambda f_h} \left[\left(x + d\cos\left(\theta_r / 2\right) \right)^2 + \left(y - d\sin\left(\theta_r / 2\right) \right)^2 \right] (2)$$
$$P_{2,-\theta_r/2} = \frac{\pi}{\lambda f_h} \left[\left(x - d\cos\left(-\theta_r / 2\right) \right)^2 + \left(y + d\sin\left(-\theta_r / 2\right) \right)^2 \right] (3)$$

where *d* is the offset distance. λ is the wavelength. f_h is the hypothetical focal length.

Then, the combination of phase function of two decentered phase plates can be written as

$$P_{com} = P_{1,\theta_r/2} + P_{2,-\theta_r/2} = -4d\pi / (\lambda f_h) \cos(\theta_r / 2)x \quad (4)$$

From Eq. (4), it is obvious that the combined phase changes linearly in x-direction when the mutual rotation angle θ_r is given. The corresponding phase gradient is then given by

$$d\varphi_{p} / dx = -4d\pi / (\lambda f_{h})\cos(\theta_{r} / 2)$$
(5)

According to Eq. (1), the prism angle is then determined as follows

$$\delta = -\arcsin\left[2d / f_h \cos(\theta_r / 2)\right] \tag{6}$$

From Eq. (6), we can know that the prism angle changes continuously with the rotation angle. In other words, the two decentered phase plates can be rotated with respect to each other to achieve a reconfigurable prism.

Fig.4 illustrates the several examples for different phase combinations of two decentered phase plates with different rotation angle. In our case, the designed frequency is 30GHz, the diameter of phase plate is D = 96 mm and the offset distance is d = 20 mm. Considering designed equivalent

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Fig. 4. Different combinations of phase distribution of two decentered phase plates operating at 30GHz with different rotation angle ($\theta_{r1} = \theta_r/2$ and $\theta_{r2} = -\theta_r/2$). (a) $\theta_{r1} = 0^0$, $\theta_{r2} = 0^0$. (b) $\theta_{r1} = 30^0$, $\theta_{r2} = -30^0$.(c) $\theta_{r1} = 60^0$, $\theta_{r2} = -60^0$. (d) $\theta_{r1} = 90^0$, $\theta_{r2} = -90^0$.

prism angle is about 15 deg, the hypothetical focal length f_h is set to 157.1 mm. The corresponding phase distribution of two decentered phase plates can be calculated by Eq. (2) and (3) with $\theta_r = 0$ respectively. The gird size is 3 mm. The maximum prism angle of ~15^o can be obtained when the mutual rotation angle is $\theta_r = 0^{\circ}$. For the mutual rotation angles of $\theta_r = 60^{\circ}$ and 120^o, the combined phase can still maintain a linear distribution with different phase gradient in the direction indicated by the white arrow, which also means the corresponding prism angles change with the rotation angle. For a mutual rotation angle of $\theta_r = 180^{\circ}$, the phase distributions of two decentered phase plates are in the complementary position. The combined phase is zero, which indicates the corresponding prism angle is zero.

B. Verification of Reconfigurable Single-element Prism

To validate the performance of proposed rotationally reconfigurable prism concept, the specific phase distribution of two decentered phase plates mentioned in Section II. A is realized by utilizing the different heights of the dielectric material [15]. Fig.5(a) shows schematic of experimental setup for verification of reconfigurable prism concept. It consists of



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Fig. 5. (a) Schematic of experimental setup for verification of reconfigurable prism concept. (b) The prototype of the proposed decentered phase plate and plano-convex hyperbolic lens with PLA plastic material.

four main parts, decentered phase plate 1 and 2, classic plano-convex lens [16] with diameter of 96 mm and focal length (f) of 56 mm and Ka-band waveguide. Here, polylactic acid (PLA) plastic material is utilized to construct the prototype of the proposed decentered phase plate and plano-convex hyperbolic lens by using 3D printing, as shown in Fig.5(b). The PLA has a relative dielectric constant of ~2.75. Like Risley prism antenna, the lens is used to convert a spherical wave into a plane wave. After the plane wave passes through two phase plates, the emerging wavefront gets tilted at a certain angle.

Due to page length limit, Fig.6 only shows four examples of the radiation performance of reconfigurable prism concept operating at 30 GHz with two decentered phase plates at different rotation angle. The rotation axis of these two phase plates is aligned to z-direction. The phase plate 1 is rotated by an angle of θ_{r1} in a counterclockwise direction, and the phase plate 2 is rotated by an angle of θ_{r2} in a clockwise direction. The Ka-band waveguide is utilized as feeding antenna with linear polarization in y-direction. From simulated 3-D radiation patterns generated by using full-wave simulation in CST Microwave Studio, it can be seen intuitively that the radiation beams deflect when two decentered phase plates have different mutual rotation angles. The corresponding simulated and measured radiation patterns for the E-plane (yoz plane) are also given respectively in Fig.6, in which a good agreement could be found. The average measured realized gain of proposed prism antenna is about 20.5 dBi, which is much higher than the feed itself. Overall, the measured realized gain values are very close to the simulation results, indicating the good gain stability of proposed prism antenna with less than ~0.9dB. It demonstrates the high realized gain can be achieved owing to the good phase transformation function of two rotating phase plates. Moreover, by rotating these two phase plates along the opposite direction with the same angle, the beam radiation beams deflect in y-direction, which acts like a prism. The measured deflection angles of beam are14.1°, 12.9°, 8.4° and 0° respectively in Fig.6 (a)-(d). Table I shows comparison of deflection angles or equivalent prism angle calculated by Eq. (6) and the measurement. Although the small differences between measurement and theoretical results occur, the overall variation trend of the measured deflection angles is consistent with the



Fig. 6. The radiation performance of reconfigurable prism concept operating at 30GHz with two decentered phase plates at different rotation angle. (a) $\theta_{r1} = 0^0$, $\theta_{r2} = 0^0$. (b) $\theta_{r1} = 30^0$, $\theta_{r2} = -30^0$.(c) $\theta_{r1} = 60^0$, $\theta_{r2} = -60^0$. (d) $\theta_{r1} = 90^0$, $\theta_{r2} = -90^0$.

 TABLE I

 EQUIVALENT PRISM ANGLE AND REALIZED GAIN COMPARISON BETWEEN THE

 THEORETICAL CALCULATION AND THE MEASUREMENT

Rotation angle	Prism angle (deg)			Realized gain (dBi)	
$(\theta_{r1}, \theta_{r2})$	Theoretical	Simulated	Measured	Simulated	Measured
(0,0)	14.8	14	14.1	22.2	21.0
(30, -30)	12.7	12	12.9	21.3	20.3
(60, -60)	7.3	7	8.4	21.2	20.1
(90, -90)	0	0	0	21.6	20.5

trend of the theoretical one. Ideally, the decentered phase plates are illuminated by an ideal plane wave generated by the plano-convex hyperbolic lens. But this ideal condition is very



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Fig. 7. The simulated 3-D radiation patterns of reconfigurable prism concept operating at 30GHz by rotating two phase plates along the same direction with the same angle (a) $\theta_{r1} = \theta_{r2} = 0^{0}$.(b) $\theta_{r1} = \theta_{r2} = 60^{0}$.(c) $\theta_{r1} = \theta_{r2} = 120^{0}$.(d) $\theta_{r1} = \theta_{r2} = 180^{0}$. (c) $\theta_{r1} = \theta_{r2} = 240^{0}$. (c) $\theta_{r1} = \theta_{r2} = 300^{0}$.

difficult to be realized. Besides, the uncertainties of measurement and fabrication errors also affect the measured angles.

C. Discussion

Because the desired phase distribution of two phase plates is achieved by changing the height of the dielectric material (PLA) in our case, an obvious disadvantage is that the phase plate is too thick. Like dielectric Fresnel lens antenna [17], the thickness of phase plates can be further reduced by the modulo 2π operation. Moreover, other techniques such as metasurface [18] and transmitarray[19, 20] can be also applied to achieve thinner phase plates. However, this letter mainly focuses on the verification of reconfigurable mechanism of prism, instead of theses fabrication techniques. Besides, in addition to being compatible with the Risley prism antenna system to enhance the scanning flexibility, the proposed reconfigurable prism concept can be also used alone to realize 2-D beam scanning. By rotating these two phase plates along the same direction with the same angle, the 2-D beam scanning can be achieved as shown in Fig.7.

III. CONCLUSIONS

In this letter, a rotationally reconfigurable single-element prism that is compatible with the typical Risley prism antenna system is validated conceptually and experimentally. It consists of a pair of rotating decentered phase plates operating at 30GHz. The equivalent prism angle can be continuously changed between 0 and 14 degrees by simply rotating these two decentered phase plates. A prototype, based on the different heights of the dielectric material, has been designed, fabricated, and measured, which demonstrates the peak realized gain of ~21 dBi can be obtained with a gain variation less than 0.9-dB within the reconfigurable angle range of 0 to 14 degree. Besides, the combination of phase distribution of two rotating phase plates can be used as a new mechanism to realize 2D beam scanning, which provides a potential and simple mechanism for beam scanning antenna systems.

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REFERENCES

- Y. J. Guo and R. W. Ziolkowsk, Advanced Antenna Array Engineering for 6G and Beyond Wireless Communications. New York, NY, USA: Wiley, 2021.
- [2] R. J. Mailloux, Phased Array Antenna Handbook, 2nd ed. Norwood, MA: Artech House, 2005.
- [3] Z. Y. Wen, Y. L. Ban, Y. Yang and Q. Wen, "Risley-Prism-Based Dual-Circularly Polarized 2-D Beam Scanning Antenna with Flat Scanning Gain," *IEEE Antennas Wireless Propag. Lett.*, vol. 20, no. 12, pp. 2412-2416, Dec. 2021.
- [4] Y. C. Zhong and Y. J. Cheng, "Generating and Steering Quasi-Nondiffractive Beam by Near-Field Planar Risley Prisms," *IEEE Trans. Antennas Propag.*, vol. 68, no. 12, pp. 7767-7776, Dec. 2020.
- [5] J. B. Wang and Y. Rahmat-Samii, "A Simplified configuration of Beam Steerable Risley Prism Antennas: Principles and Validation," *IEEE Antennas Wireless Propag. Lett.*, early access, doi: 10.1109/LAWP.2022.3182267.
- [6] M. U. Afzal and K. P. Esselle, "Steering the beam of medium-to-high gain antennas using near-field phase transformation," *IEEE Trans. Antennas Propag.*, vol. 65, no. 4, pp. 1680–1690, Apr. 2017.
- [7] N. Gagnon and A. Petosa, "Using rotatable planar phase shifting surfaces to steer a high-gain beam," *IEEE Trans. Antennas Propag.*, vol. 61, no. 6, pp. 3086–3092, 2013.
- [8] Y. Sun, J. He, C. Yuan, Q. Zhang, X. Zhao, and L. Yu, "Ku-band radial line continuous transverse stub antenna with transmit-array lens for high power microwave application," *IEEE Trans. Antennas Propag.*, vol. 68, no. 3, pp. 2050–2059, Mar. 2020.
- [9] M. U. Afzal, L. Matekovits, K. P. Esselle and A. Lalbakhsh, "Beam-Scanning Antenna Based on Near-Electric Field Phase Transformation and Refraction of Electromagnetic Wave Through Dielectric Structures," *IEEE Access*, vol. 8, pp. 199242-199253, Oct. 2020.
- [10] N. Gagnon and A. Petosa, "Using rotatable planar phase shifting surfaces to steer a high-gain beam", *IEEE Trans. Antennas Propag.*, vol. 61, no. 6, pp. 3086-3092, Jun. 2013.
- [11] H. F. Wang, Z. B. Wang, Z. H. Wu, and Y. R. Zhang, "Beam-Scanning Lens Antenna Based on Elliptical Paraboloid Phase Distribution Metasurfaces," *IEEE Antennas Wireless Propag. Lett*, vol. 18, no. 8, pp. 1562-1566, Aug. 2019.
- [12] V. F. Duma and A. L. Dimb, "Exact scan patterns of rotational Risley prisms obtained with a graphical method: multi-Parameter analysis and design," *Appl. Sci.*, vol.11, no.18, pp.8451, Sept. 2021.
- [13] H. J. D. Johnsen, J. Torgersen, and A. Aksnes, "Solar tracking using beam-steering lens arrays," *Proc. SPIE*, vol. 10758, pp. 8-19, Sept. 2018.
- [14] M. Bawart, N. Bregenzer, S. Bernet, and M. R. Marte, "Dynamic beam-steering by a pair of rotating diffractive elements," *Opt. Comm.*, vol.460, pp.125071, Apr. 2020.
- [15] H. Yi, S. W. Qu, K. B. Ng, C. H. Chan and X. Bai, "3-D Printed Millimeter-Wave and Terahertz Lenses with Fixed and Frequency Scanned Beam," *IEEE Trans. Antennas Propag.*, vol. 64, no. 2, pp. 442-449, Feb. 2016.
- [16] P. Piksa, S. Zvanovec, and P. Cerny, "Elliptic and hyperbolic dielectric lens antennas in mmwaves," *Radioengineering*, vol. 20, no. 1, pp.270-275. Apr. 2011.
- [17] S. Zhang, P. Liu and W. Whittow, "Design and Fabrication of 3-D-Printed High-Gain Broadband Fresnel Zone Lens Using Hybrid Groove-Perforation Method for Millimeter-Wave Applications," *IEEE Antennas Wireless Propag. Lett.*, vol. 21, no. 1, pp. 34-38, Jan. 2022.
 [18] K. Singh, M. U. Afzal and K. P. Esselle, "Designing Efficient
- [18] K. Singh, M. U. Afzal and K. P. Esselle, "Designing Efficient Phase-Gradient Metasurfaces for Near-Field Meta-Steering Systems," *IEEE Access*, vol. 9, pp. 109080-109093, Jul. 2021.
- [19] Z. Zhang, H. Luyen, J. H. Booske and N. Behdad, "X-band mechanically-beam-steerable lens antenna exploiting the Risley prism concept", *IET Microw. Antennas Propag.*, vol. 14, no. 14, pp. 1902-1908, Nov. 2020.
- [20] S. Liu, H. Sato and Q. Chen, "A Wideband, 1 bit Transmitarray Antenna Design With Flat Gain Response," *IEEE Trans. Antennas Propag.*, vol. 68, no. 10, pp. 7046-7055, Oct. 2020.