

Frequency Characteristics of Circuit Model of Undersea Loop Coupler Derived by Impedance Double Expansion Method

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Abstract—The impedance double expansion method (IDEM) is a circuit modeling technique for electrically small devices in the vicinity of lossy dielectrics. This paper discusses the frequency characteristics of the circuit model of an undersea loop coupler, which was previously derived by the IDEM. Comparison of the results by the circuit model and full-wave analyses indicates that the loss of the coupler in the low frequencies can be approximated only by impedance components due to eddy currents.

Keywords—Dielectric losses, equivalent circuits, method of moments, wireless power transmission.

I. INTRODUCTION

The impedance expansion method (IEM) is a circuit modeling technique for electrically small devices such as couplers for wireless power transfer (WPT) [1], [2]. The IEM is based on the Laurent series expansion of the impedance matrix in the method of moments (MoM) with respect to the complex angular frequency, and features no need for curve fitting. So far, the IEM has been applied to WPT systems in free space and in the vicinity of lossless conducting, dielectric, or magnetic scatterers [3], [4]. The impedance double expansion method (IDEM) is a further extension of the IEM to handle lossy media [5]. This paper discusses the frequency characteristics of the circuit model of undersea loop coupler, which was previously derived in [5].

II. BRIEF OVERVIEW OF IDEM

Assume that the basis functions $\mathbf{F}_i$ and $\mathbf{F}_j$ for representing the currents of the coupler are enclosed in a lossless dielectric whose exterior is a lossy dielectric of infinite volume. The self/mutual impedance Z_{ij} between $\mathbf{F}_i$ and $\mathbf{F}_j$ can be decomposed as follows:

$$Z_{ij} = Z_{ij}^{\text{fs}} + Z_{ij}^{\text{sc}}, \quad (1)$$

where Z_{ij}^{fs} is the free space component assuming that the entire space is filled with the lossless dielectric, and can be

expanded with respect to the complex angular frequency s as follows [3], [4]:

$$Z_{ij}^{\text{fs}} = \sum_{k=-1}^{\infty} s^k Z_{ij}^{\text{fs}(k)}, \quad (2)$$

where $Z_{ij}^{\text{fs}(0)} = 0$ holds in general and $Z_{ij}^{\text{fs}(-1)} = Z_{ij}^{\text{fs}(2)} = 0$ if $\mathbf{F}_i$ or $\mathbf{F}_j$ are solenoidal. On the other hand, Z_{ij}^{sc} is the scattering component, which represents the contribution of the scattered electric field from the surrounding lossy dielectric, and can be expanded as follows:

$$Z_{ij}^{\text{sc}} = \sum_{k=-2}^{\infty} s^{k/2} Z_{ij}^{\text{sc}(k/2)}, \quad (3)$$

where $Z_{ij}^{\text{sc}(-1/2)} = Z_{ij}^{\text{sc}(1/2)} = 0$ holds in general and $Z_{ij}^{\text{sc}(-1)} = Z_{ij}^{\text{sc}(0)} = Z_{ij}^{\text{sc}(3/2)} = 0$ if $\mathbf{F}_i$ or $\mathbf{F}_j$ are solenoidal. The scattering component in the form of (3) is obtained by finding the scattered electric field produced by a source current $I_j \mathbf{F}_j$ in the form of the Laurent series with respect to $s^{1/2}$ and testing it with $\mathbf{F}_i$.

III. NUMERICAL EXAMPLE

As shown in Fig. 1, loops 1 and 2, each of which consists of a perfect conducting wire (radius: 0.2 mm), are separately enclosed in pure water covers ($\varepsilon_{r1} = 80, \sigma_1 = 0$). The outside of the covers is assumed to be seawater ($\varepsilon_{r2} = 80, \sigma_2 = 4$ S/m). The current distributions of loops 1 and 2 are represented by thin-wire basis functions $\mathbf{F}_1$ and $\mathbf{F}_2$, which are uniform (i.e., solenoidal) along the wire axis, respectively.

The self- and mutual impedances between the loops are approximated by a Taylor series of finite order as follows:

$$Z_{ij} \simeq s Z_{ij}^{(1)} + s^2 Z_{ij}^{(2)} + s^{5/2} Z_{ij}^{(5/2)}, \quad (4)$$

where $Z_{ij}^{(1)}$ and $Z_{ij}^{(2)}$ have the distinct meaning of inductance and impedance due to eddy currents, respectively. In contrast,

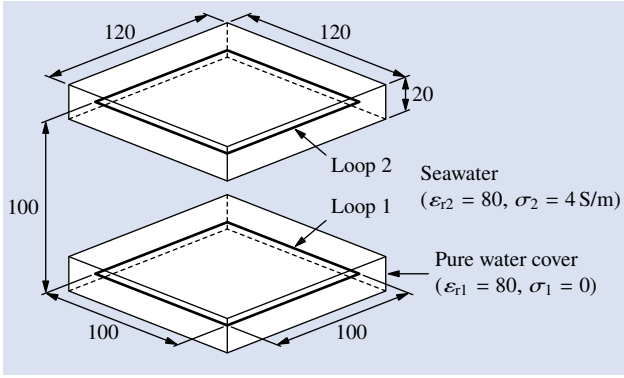


Fig. 1. Undersea loop coupler with pure water covers.

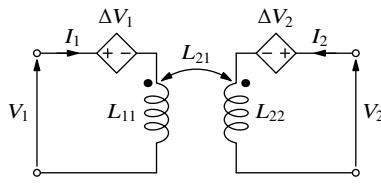


Fig. 2. Circuit model of undersea loop coupler.

the meaning of $Z_{ij}^{(5/2)}$ contains some ambiguity, but [5] concludes that it is related to radiation loss. In the following, the difference in the frequency characteristics with and without $Z_{ij}^{(5/2)}$ is focused on.

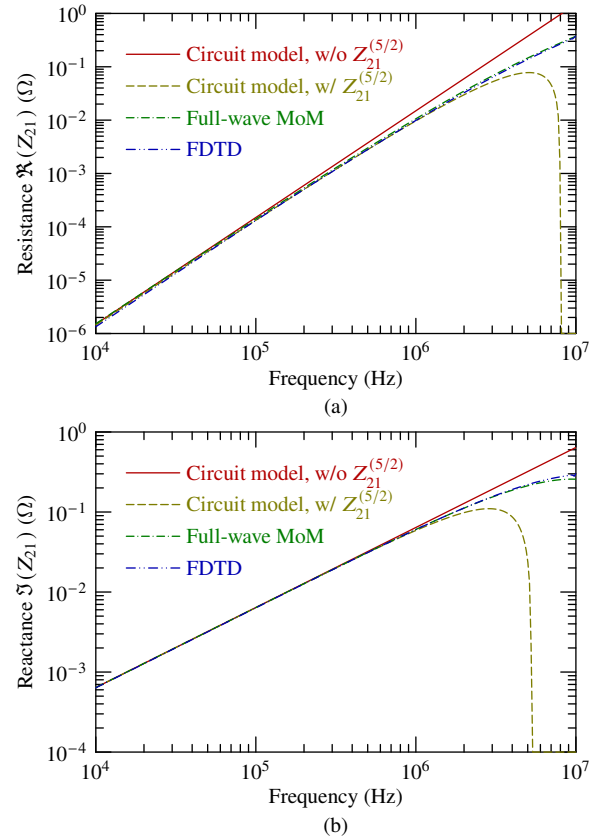
Fig. 2 shows the circuit model representing the self- and mutual impedances in (4). The self- and mutual inductances represent the impedance component proportional to s , i.e., $L_{ij} = Z_{ij}^{(1)}$. On the other hand, the dependent voltage sources represent the voltage drops caused by the higher-degree impedance components as follows:

$$\Delta V_i = \sum_{j=1}^2 \left[s^2 Z_{ij}^{(2)} + s^{5/2} Z_{ij}^{(5/2)} \right] I_j \quad (5)$$

Fig. 3 shows the frequency dependence of the mutual impedance Z_{21} , wherein the results by the circuit model with and without $Z_{21}^{(5/2)}$, full-wave MoM, and the finite-difference time-domain (FDTD) method are compared. If $Z_{21}^{(5/2)}$ is ignored, then the error is less than 10% below approximately 1.06 MHz, indicating that the loss in this frequency range is mainly due to the eddy currents. On the other hand, if $Z_{21}^{(5/2)}$ is considered, then the frequency at which the error is less than 10% rises to approximately 1.79 MHz. However, care should be taken because the results obtained by the circuit model diverge from other results at frequencies above approximately 50 MHz.

IV. CONCLUSION

This paper discusses the frequency characteristics of the circuit model of an undersea loop coupler. As a result, it is concluded that the loss of the coupler in the low frequencies can be approximated only by impedance components due to eddy currents.


 Fig. 3. (a) Real and (b) imaginary parts of the mutual impedance Z_{21} .

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REFERENCES

- [1] N. Haga and M. Takahashi, "Circuit modeling technique for electrically-very-small devices based on Laurent series expansion of self-/mutual impedances," *IEICE Trans. Commun.*, vol. E101-B, no. 2, pp. 555–563, Feb. 2018.
- [2] N. Haga and M. Takahashi, "Circuit modeling of a wireless power transfer system by eigenmode analysis based on the impedance expansion method," *IEEE Trans. Antennas Propag.*, vol. 67, no. 2, pp. 1233–1245, Feb. 2019.
- [3] N. Haga, J. Chakrothai, and K. Konno, "Circuit modeling of wireless power transfer system in the vicinity of perfectly conducting scatterer," *IEICE Trans. Commun.*, vol. E103-B, no. 12, pp. 1411–1420, Dec. 2020.
- [4] N. Haga, J. Chakrothai, and K. Konno, "Circuit modeling of a wireless power transfer system containing ferrite shields using an extended impedance expansion method," *IEEE Trans. Microw. Theory Tech.*, vol. 70, no. 5, pp. 2872–2881, May 2022.
- [5] N. Haga, J. Chakrothai, and K. Konno, "Circuit modeling of near-field coupled undersea antennas using impedance double expansion method," *IEEE Trans. Antennas Propag.*, vol. 72, no. 12, pp. 9378–9391, Dec. 2024.