Circuit Modeling of Wireless Power Transfer Systems Using Impedance Expansion Method

(Invited Paper)

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***Abstract*—The impedance expansion method (IEM) is a circuit modeling technique based on the method of moments. The IEM has been extended mainly for application to wireless power transfer systems. For example, the IEM has been extended to consider the presence of perfectly conducting and dielectric/magnetic scatterers. This paper introduces the basic concept, theoretical extension, and recent application of the IEM.**

***Index Terms*—Equivalent circuits, method of moments, wireless power transmission.**

I. Introduction

Wireless power transfer (WPT) systems generally consist of transmitting (Tx) and receiving (Rx) circuits and a wireless coupler. Since the Tx and Rx circuits are nonlinear, time-domain circuit analysis is necessary for their design. Although it is possible to model the wireless coupler using the finite-difference time-domain method to perform an integrated analysis of the entire system, it requires a significant computational cost. However, if the wireless coupler can be modeled as an electric circuit, the integrated analysis and design can be conducted efficiently [1], [2]. For this purpose, a circuit modeling technique called the impedance expansion method (IEM) has been devised and extended [3]–[6]. This paper introduces the basic concept and theoretical extension of the IEM. Additionally, the recent application of the IEM is discussed as an extension of [6].

II. Basic Concept

The IEM is based on the method of moments (MoM), which is one of numerical electromagnetic field analysis methods [7]. In the MoM, the currents on conductors are expanded into several basis functions, as shown in Fig. 1(a). Then, the matrix equation $\bar{\mathbf{Z}}\mathbf{I} = \mathbf{V}$ is yielded by means of the weighted residual method, where $\bar{\mathbf{Z}}$ is the impedance matrix, $\mathbf{I}$ is the current coefficient vector, and $\mathbf{V}$ is the voltage coefficient vector. Since the matrix equation has the same form as the circuit equations in the mesh analysis, the equivalent expression in Fig. 1(b) holds for the discretized voltages and currents.

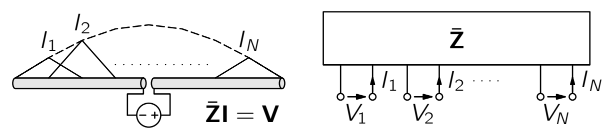

Fig. 1. (a) Expanded current and (b) equivalent network.

$$\bar{\mathbf{Z}} = s^{-1}\bar{\mathbf{Z}}^{(-1)} + s\bar{\mathbf{Z}}^{(1)} + s^2\bar{\mathbf{Z}}^{(2)} + s^3\bar{\mathbf{Z}}^{(3)} + s^4\bar{\mathbf{Z}}^{(4)} + \cdots$$

Fig. 2. Impedance matrix expanded into the Laurent series.

In the IEM, the impedance matrix $\bar{\mathbf{Z}}$ is expanded into the Laurent series with respect to the complex angular frequency $s = j\omega$ as shown in Fig. 2, where the impedance components proportional to s^{-1} and s correspond to the reactances of capacitors and inductors, respectively, and those proportional to s^2 and s^4 correspond to the radiation resistances of infinitesimal dipoles and loops, respectively [3], [4]. These radiation resistances can be represented by dependent voltage sources, or they can be approximated only by passive elements.

By solving a generalized eigenvalue problem $\bar{\mathbf{Z}}^{(-1)}\mathbf{I} = \omega^2\bar{\mathbf{Z}}^{(1)}\mathbf{I}$, which represents the series resonance condition, modal currents can be obtained. By expanding the currents on the conductors with a small number of modal currents whose resonant frequencies are close to the operating frequency, the number of unknowns in the circuit model can be reduced [4].

III. Consideration of Neighboring Scatterer

The IEM has been extended to consider the scattering by perfect conductors [5] and dielectric/magnetic bodies [6]. As shown in Fig. 3, if there is a conducting or dielectric/magnetic scatterer in the vicinity of basis functions $\boldsymbol{F}_i$ and $\boldsymbol{F}_j$, the self-/mutual impedance between $\boldsymbol{F}_i$ and $\boldsymbol{F}_j$ can be decomposed as $Z_{ij} = Z_{ij}^{\mathrm{fs}} + Z_{ij}^{\mathrm{sc}}$, where Z_{ij}^{fs} is the free-space component, which

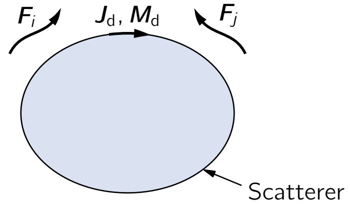

Fig. 3. Basis functions $\boldsymbol{F}_i$ and $\boldsymbol{F}_j$ in the vicinity of a scatterer.

is the self-/mutual impedance in the absence of the scatterer, and Z_{ij}^{sc} is the scattering component, which is induced by the scattered field and can be obtained by the following procedure:

1) The equivalent electromagnetic currents $\boldsymbol{J}_{\mathrm{d}}$ and $\boldsymbol{M}_{\mathrm{d}}$ on the surface of the scatterer (or the surface current $\boldsymbol{J}_{\mathrm{c}}$ if the scatterer is perfectly conducting) by $\boldsymbol{F}_j$ is obtained in the form of the Laurent series with respect to s.
2) The dot product of $\boldsymbol{F}_i$ and the scattered electric field produced by $\boldsymbol{J}_{\mathrm{d}}$ and $\boldsymbol{M}_{\mathrm{d}}$ is integrated.

As a result, Z_{ij}^{sc} can be obtained as the Laurent series with respect to s, but it converges only when the dimension of the scatterer is less than 0.25 to 0.3 times the wavelength [5], [6].

IV. WPT System Containing Ferrite Shields

The extended IEM has been applied to a WPT system containing ferrite shields, which is shown in Fig. 4 [6]. The radius and conductivity of the wires were assumed to be 0.4 mm and 58 S/m, respectively. The Tx and Rx sides were arranged in mirror symmetry. Whereas the circuit model in [6] contains dependent voltage sources to represent the higher-order impedance components, this paper newly considers a circuit model approximated only by passive elements, which is shown in Fig. 5 and can be analyzed with versatile circuit simulators. Assuming uniform current distributions of the coils and using the extended IEM, the self- and mutual impedances between port 1 (Tx side) and port 2 (Rx side) are obtained in the following form:

$$Z_{ij} = sZ_{ij}^{(1)} + s^3 Z_{ij}^{(3)} + s^4 Z_{ij}^{(4)} + \cdots, \quad i, j = 1, 2, \tag{1}$$

where the self-impedance components due to the finite conductivity of the wires, which are denoted by Z_1 and Z_2 in Fig. 5, are not included. On the other hand, the self- and mutual impedances between ports 1 and 2 in Fig. 5, excluding Z_1 and Z_2, can be expanded as follows:

$$\begin{aligned} Z_{11} = {} & s(L_1 + L_3) - s^3(L_1^2 C_1 + M_{21}^2 C_2) \\ & + s^4(L_1^2 C_1^2 R_1 + M_{21}^2 C_2^2 R_2) + \cdots, \end{aligned} \tag{2}$$

$$\begin{aligned} Z_{21} = {} & s(M_{21} + M_{43}) - s^3(L_1 C_1 + L_2 C_2) M_{21} \\ & + s^4(L_1 C_1^2 R_1 + L_2 C_2^2 R_2) M_{21} + \cdots, \end{aligned} \tag{3}$$

$$\begin{aligned} Z_{22} = {} & s(L_2 + L_4) - s^3(M_{21}^2 C_1 + L_2^2 C_2) \\ & + s^4(M_{21}^2 C_1^2 R_1 + L_2^2 C_2^2 R_2) + \cdots. \end{aligned} \tag{4}$$

Here, the Tx and Rx sides are assumed to be symmetric, i.e., $Z_{11} = Z_{22}, L_1 = L_2, L_3 = L_4, C_1 = C_2$, and $R_1 = R_2$,

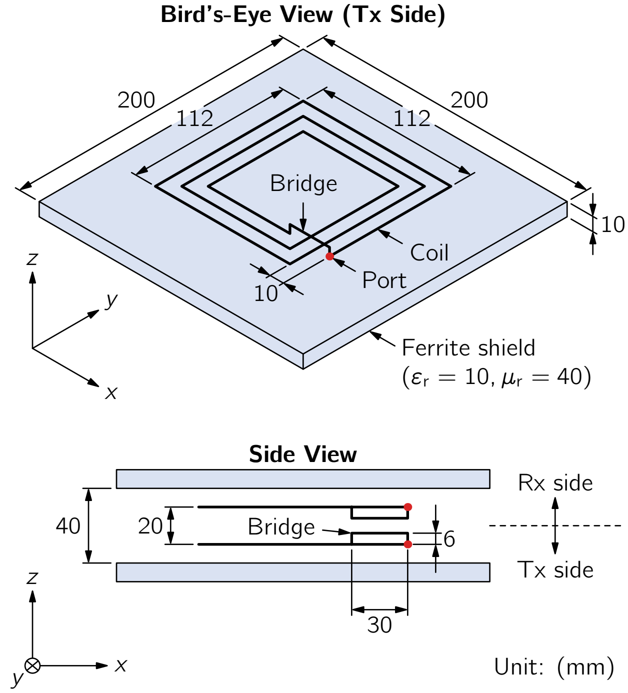

Fig. 4. WPT system containing ferrite shields [6].

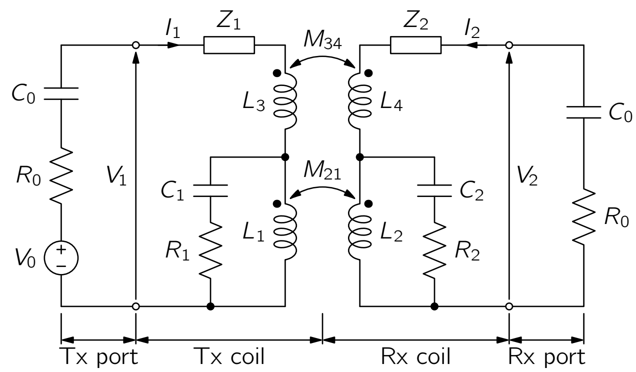

Fig. 5. Circuit model of the WPT system approximated only by passive elements.

and the terms proportional to s^4 in (2) and (3) are assumed to coincide with $Z_{11}^{(4)}$ and $Z_{21}^{(4)}$, respectively, i.e.,

$$(L_1^2 + M_{21}^2) C_1^2 R_1 = Z_{11}^{(4)}, \tag{5}$$

$$2 L_1 M_{21} C_1^2 R_1 = Z_{21}^{(4)}. \tag{6}$$

In [4], the equations corresponding to (5) and (6) were solved assuming $Z_{ij}^{(4)2} \simeq Z_{ii}^{(4)} Z_{jj}^{(4)}$, but this approximation is not used here. (5) and (6) can be solved for M_{21} and R_1 as follows:

$$M_{21} = \frac{L_1 Z_{21}^{(4)}}{Z_{11}^{(4)} + \sqrt{Z_{11}^{(4)2} - Z_{21}^{(4)2}}}, \tag{7}$$

$$R_1 = \frac{Z_{11}^{(4)} + \sqrt{Z_{11}^{(4)2} - Z_{21}^{(4)2}}}{2 L_1^2 C_1^2}, \tag{8}$$

TABLE I
CIRCUIT PARAMETERS WITH AND WITHOUT THE FERRITE SHIELDS

	w/ shields	w/o shields
R_0 [Ω]	50.00	24.00
C_0 [pF]	274.6	371.9
$L_1 = L_2$ [μH]	1.896	1.097
M_{21} [μH]	1.632	1.081
$L_3 = L_4 = -M_{43}$ [nH]	439.0	510.9
$C_1 = C_2$ [fF]	7.809	4.558
$R_1 = R_2$ [kΩ]	5.010	33.66
$Z_1 = Z_2$ [mΩ]	$311.4 + j301.4$	

which means that the choice of L_1 and C_1 remains arbitrary.

The inductances L_1, L_3, and M_{43} were specified so that the terms proportional to s in (2) and (3) coincide with $Z_{11}^{(1)}$ and $Z_{21}^{(1)}$, respectively, i.e., $L_1 + L_3 = Z_{11}^{(1)}$ and $M_{21} + M_{43} = Z_{21}^{(1)}$. Although the relationship between L_1 and M_{21} is constrained by (7), there remains a degree of freedom in the ratio of L_1 to L_3. Here, the inductances were determined by adding a new constraint $L_3 = -M_{43}$, which means that L_1 was set to the largest possible value.

Then, C_1 is determined so as to minimize the residual sum of squares (RSS) in the impedance components proportional to s^3, which is defined as follows:

$$\mathrm{RSS} = \left[Z_{11}^{(3)} + \left(L_1^2 + M_{21}^2\right) C_1\right]^2 + \left[Z_{21}^{(3)} + 2L_1 M_{21} C_1\right]^2. \quad (9)$$

C_1 that minimizes the RSS can be expressed as follows:

$$C_1 = -\frac{\left(L_1^2 + M_{21}^2\right) Z_{11}^{(3)} + 2L_1 M_{21} Z_{21}^{(3)}}{L_1^4 + 6L_1^2 M_{21}^2 + M_{21}^4}. \quad (10)$$

Table I summarizes the determined circuit parameters with and without the ferrite shields, where the port impedance R_0 and matching capacitance C_0 are chosen to be appropriate values depending on the presence or absence of the shields. Additionally, the value of $Z_1 = Z_2$ is for 6.78 MHz, and it differs for other frequencies.

Fig. 6 shows the frequency dependencies of the transmission coefficient $|S_{21}|$ and radiation loss P_r with and without the ferrite shields, where the results by the circuit model and full-wave MoM are compared. It can be seen that the broadband transmission characteristics and reduced radiation loss owing to the shields are adequately reproduced in the circuit model.

V. CONCLUSION

In this paper, the basic concept and theoretical extension of the IEM were briefly summarized. Then, a circuit model approximating a WPT system containing ferrite shields only by passive elements was proposed and validated. The IEM will be further extended as required by practical applications.

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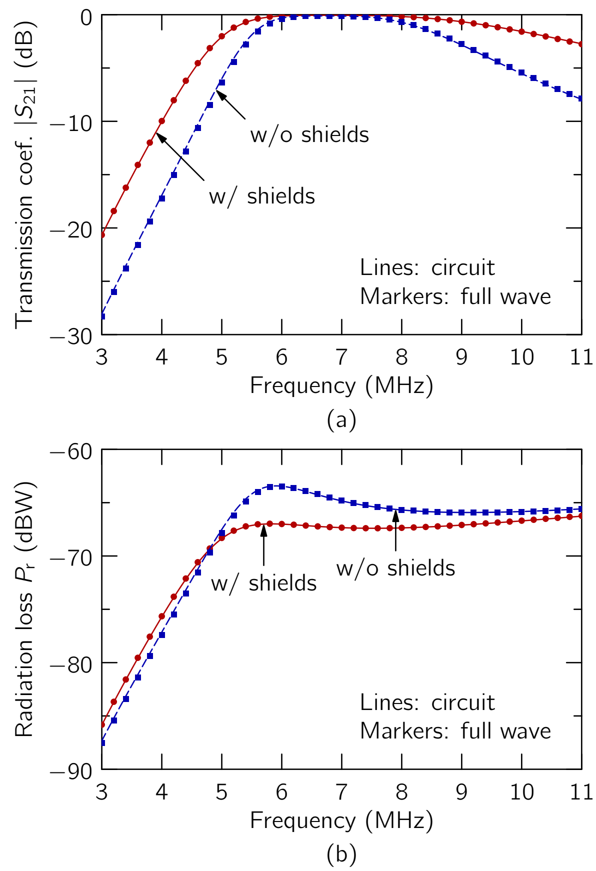

Fig. 6. Frequency dependencies of (a) transmission coefficient $|S_{21}|$ and (b) radiation loss P_r with and without the ferrite shields.

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