

A Study on Cluster-Centric Cell-Free Massive MIMO System

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Abstract— Cell-free massive multiple-input multiple-output (CF-mMIMO) systems have been attracting a great attention as a promising architecture for the beyond fifth-generation (B5G) systems. CF-mMIMO is designed to deploy a large number of base station (BS) antennas to accommodate a relatively small number of mobile users. Considering that the computational complexity and the fronthaul capacity are limited in any practical system, scalability is the key to making CF-mMIMO practical. Thus, a user-centric (UC) approach is taken, i.e., each user is served by a certain number of antennas with high channel gain (called the antenna-set). Such a CF-mMIMO is called UC CF-mMIMO in this paper. However, as the user density becomes higher, the antenna-sets tend to overlap at a higher probability, and accordingly, severer interference will occur among the neighborhood users, thereby degrading the transmission performance of UC CF-mMIMO. To mitigate this severer interference problem, in this paper, we propose to group the neighborhood users as the user-cluster and to apply multi-user MIMO (MU-MIMO) technology to each user-cluster. This CF-mMIMO is called cluster-centric (CC) CF-mMIMO. In CC CF-mMIMO, user-clusters are first formed and a set of antennas is associated with each user-cluster for performing MU-MIMO communication in parallel. We evaluate the link capacity and the user fairness by computer simulation to reveal that the proposed CC CF-mMIMO outperforms the UC CF-mMIMO.

Keywords—Cell-free massive MIMO, B5G, user-clustering, scalable implementation, signal processing

I. INTRODUCTION

The massive multiple-input-multiple-output (mMIMO) [1], [2] is considered as the most effective technology to significantly improve the spectrum efficiency (SE) of mobile communication systems. According to the deployment manner of base station (BS) antennas, the mMIMO system can be classified into two categories: centralized mMIMO which employs a large-scale antenna array at the BS [1], and distributed mMIMO which distributes a large number of antennas over the BS service area. The distributed mMIMO can provide a uniform service quality over the entire BS coverage area [3].

Recently, as a special realization of distributed massive MIMO, a so-called cell-free massive MIMO (CF-mMIMO) system has been attracting a great attention [4]–[6]. In CF-mMIMO, a large number of antennas are distributed over a wide communication service area to serve a relatively small number of users, rather than dividing the service area into a number of cells [4], [5]. Thanks to the information sharing among all antennas, CF-mMIMO can achieve a higher diversity gain and a higher interference suppression capability [4]. However, owing to the coordination between a large number of antennas, scalability is an enormous challenge for

CF-mMIMO implementation due to the prohibitively high computational complexity of signal processing and fronthaul capacity requirement. Therefore, in [6], a user-centric (UC) approach was proposed to realize the scalable implementation of CF-mMIMO. Specifically, in UC CF-mMIMO, each user is served by only a set of high channel gain antennas (called antenna-set). Since the antenna-sets determined for different users are inevitably overlapped especially in the dense user environment which leads to an inter-user-interference (IUI), the multi-user precoding/postcoding-based interference mitigation is supposed to be taken for each user's signal transmission. Accordingly, in [6], the partial minimum mean-squared-error (MMSE) was proposed to mitigate the IUI from a part of users taking into account scalability. However, in the above scenario, as the user density becomes higher, the transmission performance tends to degrade due to severer IUI.

In this paper, to further improve the transmission performance of scalable CF-mMIMO, we propose to incorporate the multi-user MIMO (MU-MIMO) technology into UC CF-mMIMO, which means that we group neighborhood users into a user-cluster and apply MU-MIMO signal transmission to spatial-multiplex those users without causing IUI. We call this approach the cluster-centric (CC) approach in this paper. In CC CF-mMIMO, user-clusters are first formed and then, a set of antennas is associated with each user-cluster. The set of antennas for each user-cluster is allowed to overlap with those of other user-clusters. We derive the pre/postcoding weight and the SINR expression and confirm by computer simulation that the proposed CC CF-mMIMO is superior to UC CF-mMIMO.

The rest of this paper is organized as follows. In Section II, we introduce the system model and the details of user-clustering and antenna association progress for the CC approach compared with the UC approach. In Section III, we describe the transmission model of scalable CF-mMIMO based on zero-forcing (ZF) multiplexing [7], [8] and derive the generalized signal-to-interference-plus-noise-ratio (SINR) expression which is applicable to both UC and CC approaches. In Section IV, the computer simulation results are presented to demonstrate the advantages of the proposed CC approach. In the final Section V, we give some conclusions and future studies.

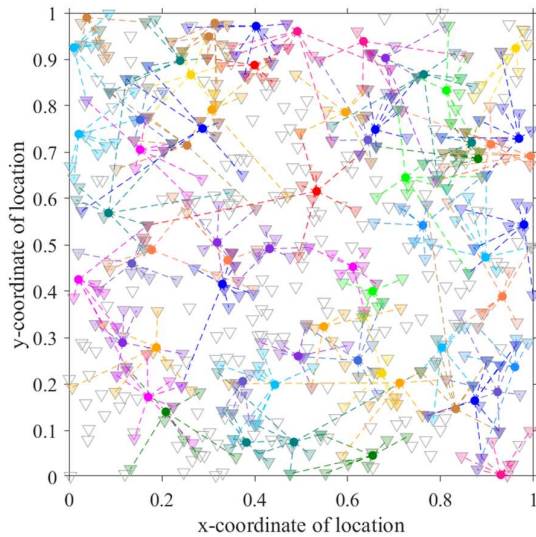
II. SYSTEM MODEL

In this paper, a CF-mMIMO system model with a normalized 1×1 square-shaped service area is considered, so that it can be scaled to any real size of the service area. In the service area, A distributed antennas are deployed and the BS which has a central processing unit (CPU) coordinates all A distributed antennas to serve U single-antenna users. We

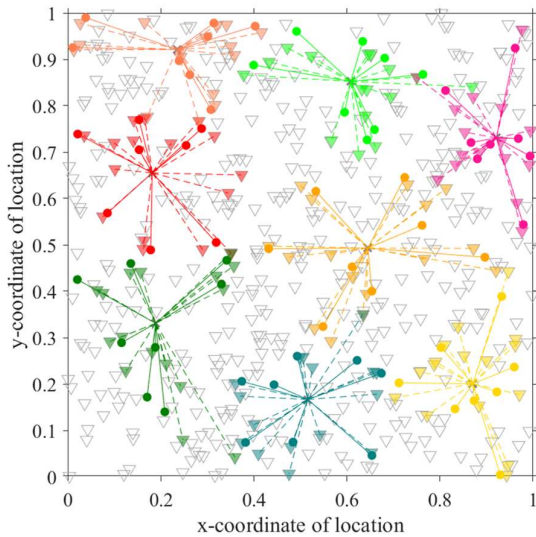
assume that users and antennas are randomly distributed within the service range. Also, we assume that perfect channel information is available at both the BS and user sides.

To implement scalable CF-mMIMO, we take the CC approach and divide users into K user-clusters based on their location information. To facilitate comparison with the UC approach, user-clusters with the same size are constructed by the modified K-means algorithm [9], [10]. The number of users in the k th cluster is denoted as U_k , $k=1, \dots, K$, where $K=U/U_k$ (for simplicity, we assume that U is an integer multiple of U_k). We denote the user-set $\mathcal{S}_k \subset \{1, \dots, u, \dots, U\}$ to present users belonging to the k th cluster, then, $|\mathcal{S}_k| = U_k$. Moreover, the user-clusters are non-overlapped, i.e., $\mathcal{S}_k \cap \mathcal{S}_i = \emptyset$. It should be noted that, in the UC approach [6], each user can be treated as a single-user cluster, i.e., $K=U$.

▽: antenna ○: user ×: cluster centroid
—: user clustering ---: antenna association



(a) User-centric ($K=U$, $A_k=8$)



(b) Cluster-centric ($K=8$, $A_k=16$)

Fig. 1. An example of cluster structure with antenna association ($A=512$, $U=64$).

After the user-clustering, a set of distributed antennas is associated with each user-cluster. The antenna-sets for different user-clusters are allowed to be overlapped. For the UC approach, A_k (number of antennas in the k th cluster) high gain antennas are associated with each user. For the proposed CC approach, since MU-MIMO is implemented in each cluster, each user in the cluster needs to be fairly associated with high gain antennas, that is, the same number of antennas, i.e., A_k/U_k antennas (A_k is assumed to be an integer multiple of U_k) are associated with each user in the same cluster. Then, in general, the antenna-set associated with the k th cluster is denoted by $\mathcal{M}_k \subset \{1, \dots, a, \dots, A\}$ with $|\mathcal{M}_k| = A_k$.

Later, the examples of antenna association for the UC and CC approaches are illustrated in Fig. 1. In Fig. 1 (a), each user is represented by different colors as the single-user cluster. The antenna is colored according to the color of the associated user, and the antenna that is not assigned is not colored. In this paper, to make the comparison between UC and CC approaches clear, we set $A_k=8$ in each single-user cluster in the UC approach. We can see that the antenna-sets of different users overlap. Similarly, in Fig.1 (b), users belonging to the same cluster and their associated antennas are highlighted by the same color. We can see that the antenna association result of CC is more compact when the total number of antennas is larger than that of users (i.e., $A=512$, $U=64$), and the clustering division is distinct. Just as the number of associated antennas of each user in the UC approach is fixed, the number of users in each cluster is also fixed to 8 in CC approach. Accordingly, the size of the associated antenna-set in each cluster in the CC approach is determined as 16 with the assumption of the same computational complexity required for UC and CC approaches. We will present this in detail in the following Section III.

III. ZF-BASED MU-MIMO TRANSMISSION MODEL

In this section, we present the ZF-based MU-MIMO transmission scenario for scalable CF-mMIMO. Here, we derive the expressions for the pre/postcoding weight vector and those for the achievable SINR for CC CF-mMIMO. As we mentioned above, the UC approach is regarded as a special case of the CC one (i.e., single-user cluster), so the formulas deduced below are also applicable to the UC approach. First, inspired by [6], we define the antenna association matrix for the k th cluster as $\mathbf{D}_k \in \mathbb{C}^{A \times A} = \text{diag}(d_1, \dots, d_a, \dots, d_A)$, where $d_a=1$ indicates the a th antenna is associated with the k th cluster, otherwise, $d_a=0$. Then, based on that, assuming the u th user belongs to the k th cluster, i.e., $u \in \mathcal{S}_k$, we derive the uplink received signal expression after postcoding for the u th user as

$$y_u = \sum_{i=1}^K \sum_{v \in \mathcal{S}_i} \mathbf{w}_u \mathbf{D}_k \mathbf{h}_v \sqrt{P_v} s_v + \mathbf{w}_u \mathbf{D}_k \mathbf{n}_u, \quad (1)$$

where \mathbf{h}_u , \mathbf{w}_u , P_u , s_u , \mathbf{n}_u are the uplink channel between all the antennas to the u th user, postcoding vector for the u th user, transmit power for the u th user, transmit signal for the u th user and noise vector at service antennas for the u th user, respectively. To make our investigation close to the reality, we characterize the MIMO channel assuming the well-known propagation channel model consisting of path loss, shadowing loss, and Rayleigh fading.

In precise, the uplink ZF postcoding weight vector for the u th user can be represented as

$$\mathbf{w}_u = \begin{cases} \mathbf{h}_u^H \mathbf{D}_k \left(\sum_{i=1}^K \sum_{v \in \mathcal{S}_i} \mathbf{D}_k \mathbf{h}_v \mathbf{h}_v^H \mathbf{D}_k \right)^\dagger, \text{full ZF} \\ \mathbf{h}_u^H \mathbf{D}_k \left(\sum_{v \in \mathcal{N}_k} \mathbf{D}_k \mathbf{h}_v \mathbf{h}_v^H \mathbf{D}_k \right)^\dagger, \text{partial ZF} \end{cases}, \quad (2)$$

where \mathbf{A}^\dagger denotes the Moore–Penrose inverse of matrix \mathbf{A} . In (2), if all the users in the system are considered for weight calculation for a cluster, we called it full ZF. Then, by contrast, partial ZF is defined as the case of only a part of users treated as the considered users for weight calculation. Here, \mathcal{N}_k is the user set to be considered in the partial ZF weight calculation for the k th cluster. In the case of UC approach, $\mathcal{N}_{u=k}$ consists of the u th user of interest and interfering users whose antenna-sets overlap that of the u th user. That is, $\mathcal{N}_{u=k} = \{j : \mathbf{D}_u \mathbf{D}_j \neq \mathbf{0}_A\}$ [6]. Then, on the other hand, in order to ensure that the channel estimation complexity is the same for both UC and CC approaches, in this paper, we define the set of interfering users for a cluster in the CC approach as the union of the sets of interfering users determined for each user in that cluster in the UC approach. Namely, $\mathcal{N}_k = \bigcup_{u \in \mathcal{S}_k} \mathcal{N}_u$.

As presented in (2), we express the above formula from the antenna dimension (based on the uplink channel vector) because it can freely adjust the number of users considered in ZF weight calculation. Moreover, the computational complexity of the above antenna dimension-based matrix inversion is fixed once the associated antennas are decided and its required computational complexity is relatively small compared with the user dimension-based matrix inversion in the dense user environment because the number of users considered in ZF weight calculation is usually large in that case. Since the computational complexity of matrix inversion in (2) is $O(A_k^3)$ [6], [8], the total required computational complexity for the system is $K \times O(A_k^3)$. It suggests that the UC approach requires U/K times larger number of matrix inversion operations than the CC approach when the same number of antennas is associated with each cluster. In other words, more antennas can be associated with each cluster in the CC approach under the same computational complexity of matrix inverse required in the UC approach. In precise, since the number of clusters in the UC approach is equal to the number of users in the system, and the number of antennas in each cluster is fixed to 8, it can be deduced that in the CC approach when the number of users in the cluster is set to 8, $\sqrt[3]{8} \times 8 = 16$ antennas is allowed to be associated with each cluster.

$$\text{SINR}_u = \begin{cases} \frac{P_u |\mathbf{w}_u \mathbf{D}_k \mathbf{h}_u|^2}{\sum_{i=1}^K \sum_{v \in \mathcal{S}_i} P_v |\mathbf{w}_v \mathbf{D}_k \mathbf{h}_v|^2 - P_u |\mathbf{w}_u \mathbf{D}_k \mathbf{h}_u|^2 + \|\mathbf{w}_u\|_F^2}, \text{UL} \\ \frac{P_u |\mathbf{h}_u^T \mathbf{D}_k \mathbf{w}_u|^2}{\|\mathbf{w}_u\|_F^2}, \text{DL} \end{cases} \quad (3)$$

Note that according to the symmetry nature of uplink and downlink channels, the downlink channel vector and precoding vector for the u th user are the transposition of \mathbf{h}_u

and \mathbf{w}_u , respectively. When the power spectral density of the user signal and that of the additive white Gaussian noise (AWGN) have zero mean and unity variance, the SINR of the u th user can be derived as in (3).

IV. NUMERICAL RESULTS

In this section, we evaluate and discuss the performance of UC and CC approaches in terms of user capacity, sum capacity, and user fairness, respectively obtained by using (4), (5), and (6). In (6), the well-known Jain's fairness index [11] is used to represent the user fairness based on user capacity.

$$C_u = \log_2(1 + \text{SINR}_u), \quad (4)$$

$$C_{\text{sum}} = \sum_u C_u, \quad (5)$$

$$\text{FI} = \left[\sum_u C_u \right]^2 / \sum_u C_u^2. \quad (6)$$

Assuming a fixed randomly generated antenna location pattern as illustrated in Fig. (1), we randomly change the user location pattern 100 times to implement a Monte-Carlo simulation. For each user location pattern, the shadowing loss and Rayleigh fading gain are randomly generated for each user-to-antenna channel according to log-normal distribution and zero-mean complex-Gaussian distribution, respectively. Then, the antenna-cluster association is performed (note that in the UC case, only one user exists in each cluster). After that, full and partial ZF for UC and CC approaches are carried out, respectively, to compute the user capacity, sum capacity, and user fairness for obtaining their cumulative distribution functions (CDFs). In addition, the transmit power for each user is the same and is represented by the normalized transmit signal-to-noise ratio (SNR) which is defined as the received SNR when the transmitter-receiver distance is equal to the side length of the normalized 1×1 square-shaped area. The simulation parameter setting is shown in Table I.

TABLE I. SIMULATION SETTING

Parameter	Value/State	
	User-centric	Cluster-centric
Number of distributed antennas (A)	512	
Number of users (U)	32, 64, 128, 256, 512	
Number of users per cluster (U_k)	1	8
Number of clusters (K)	U	$U/8$
Number of antennas per cluster (A_k)	8	16
Number of times of user location generations	100	
Path loss exponent	3.5	
Shadowing standard deviation [dB]	8	
Fading type	Rayleigh	
Transmit SNR per user (P) [dB]	-30	

Fig. 2 plots the values of user capacity, the sum capacity, and the user fairness at the probability in their CDFs equal to 50% under the different numbers of users in downlink and uplink cases. Here, the blue triangle and red circle marks represent UC and CC approaches, respectively, while the solid lines and dashed lines represent full and partial ZF transmission, respectively. First of all, comparing the solid lines, it can be clearly demonstrated that the proposed CC approach achieves a higher capacity and user fairness than the UC approach in the full ZF case. Then, in the partial ZF case, the capacity and fairness reduce for both UC and CC approaches because only a part of interfering users is considered in the pre/postcoding weight vector. But, still, the proposed CC approach outperforms the UC one. Despite the

differences in the specific values, the results observed above hold both in the uplink and downlink. In addition, we also point out that the difference in user capacity between full and partial ZF increases and then decreases as the number of users increases. This difference is most obvious when $U=64$ and 128 for UC, and when $U=64$ for CC. Next, we use some examples to analyze the reasons for the above results.

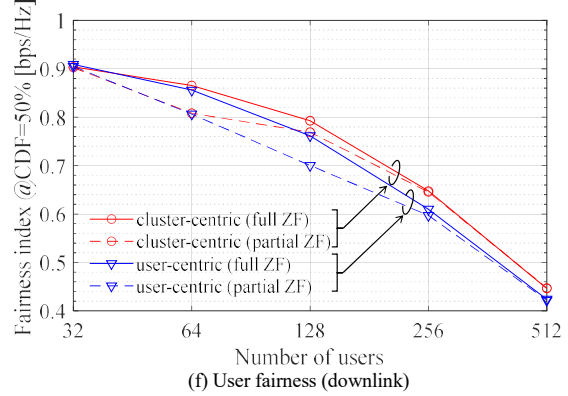
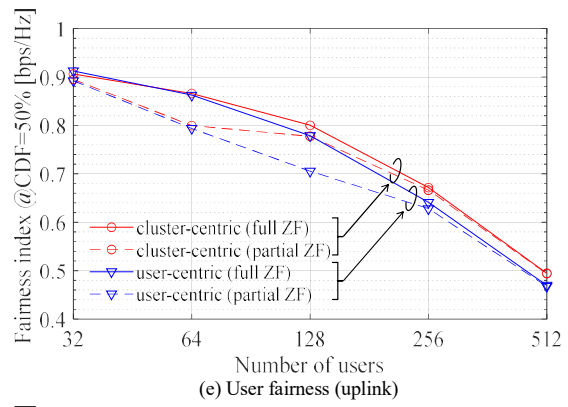
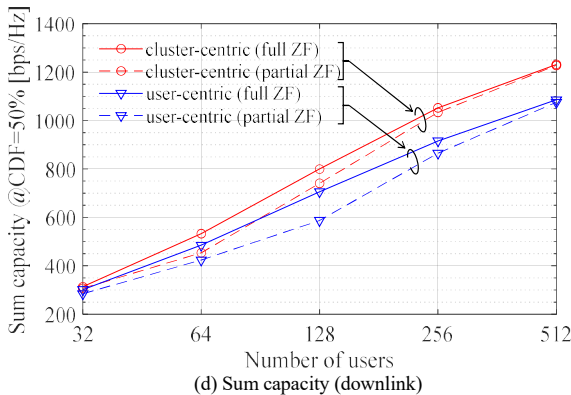
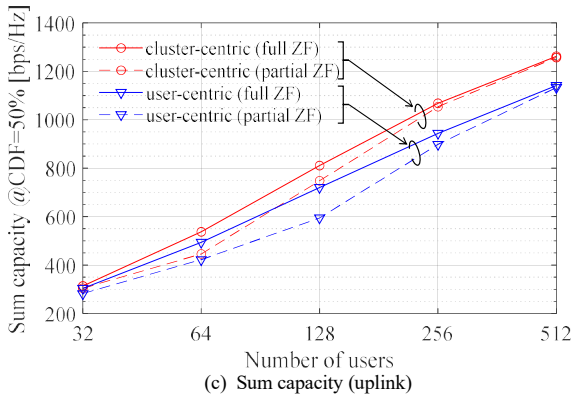
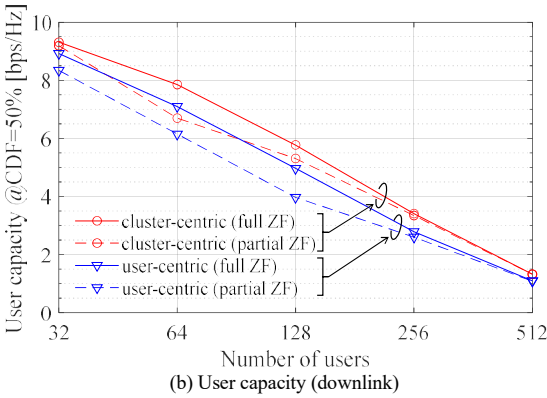
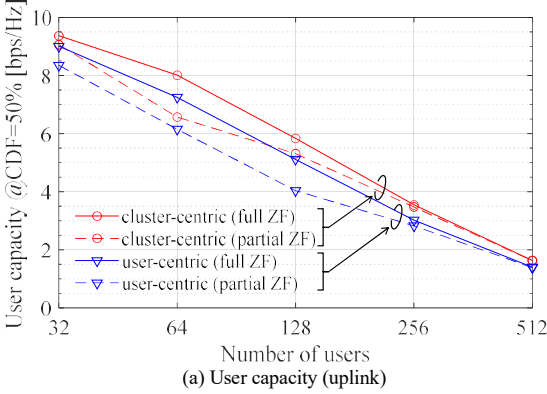
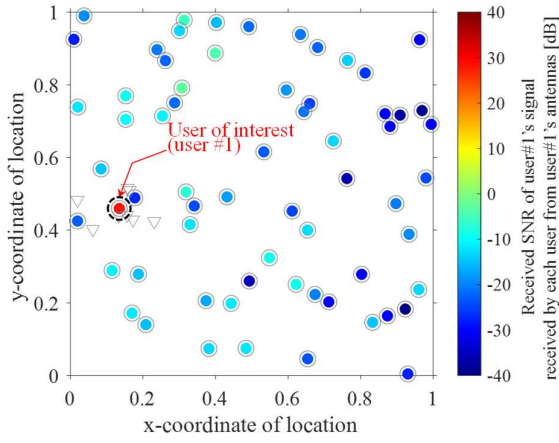


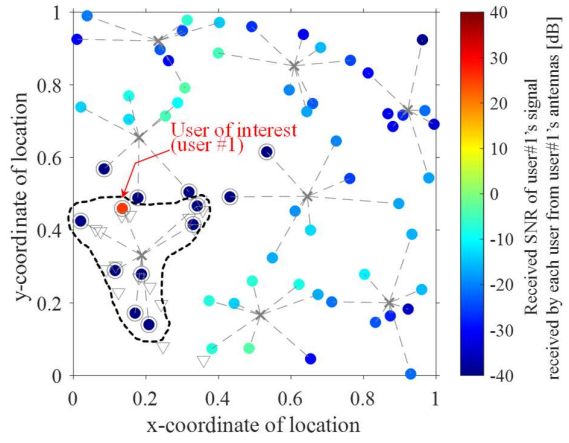
Fig. 2. Comparison of UC and CC approaches with full and partial ZF in terms of user capacity, sum capacity, and user fairness.

In Fig. 3, assuming the same antenna and user pattern as in Fig. 1, we plot the heatmap of the received SNR of user#1 (user of interest)'s signal received by each user from the antennas associated with the cluster to which user#1 belongs in downlink transmission. Therefore, the received signal at user#1 in the heat map is its own expected signal, while the signals at other users are the interference caused by user#1. Here, we also use circles and triangles to represent users and antennas respectively. For the convenience of analysis, we only mark the antennas associated with the cluster to which user#1 belongs and indicate the received SNR level of each user at its position in blue to red. Also, for the sake of clarity, the black dashed line is drawn to indicate the cluster to which user#1 belongs. The members of each cluster are connected to the cluster centroid by dotted lines. At the same time, in order to facilitate the comparison between full ZF and partial ZF, we use a larger gray circle to mark the users considered for user#1's weight calculation.

By comparing Fig. 3 (a) and (b), we can see that in the CC approach, the interference intensity caused by user#1 to other users is generally lower than that in the UC approach, especially for users in the same cluster as user#1. It has two key factors. First, more antennas are associated in each cluster in the CC approach which provides a higher degree of freedom for generating null beams to mitigate more interference by ZF-based pre/post coding. Second, fair antenna association for each user in the cluster is carried out in the CC approach based on the MU-MIMO criterion. This brings the capability for generating null beams to perfectly cancel the interference between users in the same cluster and provides high user fairness due to the fact that each user can have the same number of high gain antennas to guarantee its appreciable signal strength.

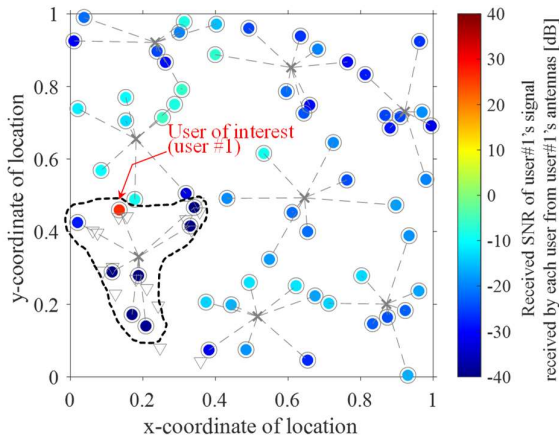


(a) User-centric (full ZF)

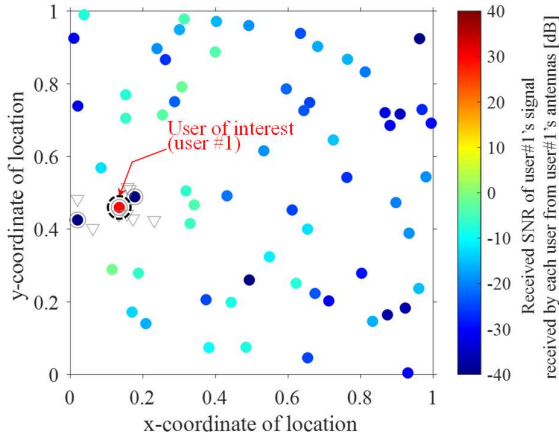


(d) Cluster-centric (partial ZF)

Fig. 3. Heatmap of received SNR of user#1 at each user in downlink ($U=64, A=512$, user and antenna distribution assumed as in Fig. 1).



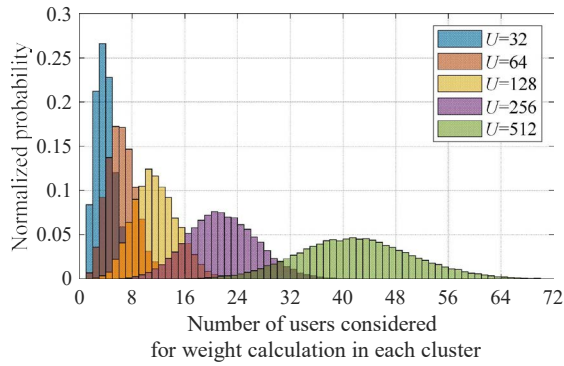
(b) Cluster-centric (full ZF)



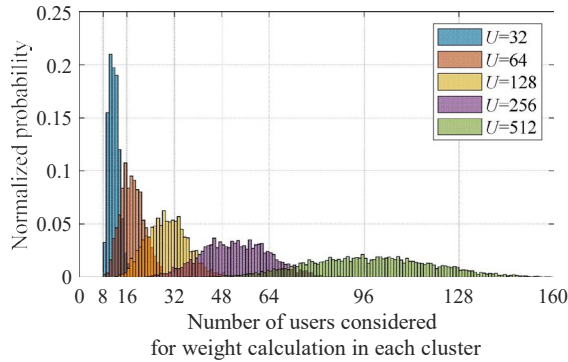
(c) User-centric (partial ZF)

Then, comparing Fig. 3 (a) with (c) and (b) with (d), respectively, we can see that only a smaller number of users (two users outside the cluster in the UC approach and five users outside the cluster in the CC approach) are considered as interfering users for ZF weight calculation, so the capacity performance of partial ZF is lower than that of full ZF. Together with Fig. 1, we can explain when the number of users is relatively small e.g., $U=64$ while $A=512$, the probability of overlapping the antenna sets in the UC approach is very small although some of the users whose antenna sets are not overlapped with a user of interest give strong interference to the user of interest. On the other hand, as already mentioned earlier, the set of interfering users to be considered in the weight calculation in the CC approach for a cluster is defined as the union of the sets of interfering users to be considered in the weight calculation in the UC approach for each user in that cluster. Therefore, more interference can be mitigated in the CC approach compared to the UC approach, as clearly seen by the comparison of Fig. 3 (c) and (d). Precisely, in the CC approach, the interference from user#1 is perfectly eliminated in a larger range, and the interference strength of user#1 is also lower for the remaining areas. The reason for this is similar to what we analyzed for the full ZF case earlier.

Through the above analysis of the heat maps, we can conclude that the gap between partial ZF and full ZF is widened in the medium number of users (e.g., $U=64$ in Fig. 3) because it does not effectively judge the interfering users considered for weight calculation, remaining a lot of interference. However, as the user density increases, the probability that the associated antenna-sets of different clusters overlap is bound to increase, so the judgment of interfering users in ZF weight calculation in partial ZF will be more effective to obtain a better interference mitigation. Therefore, from Fig. 2 (a), we can see that the difference between partial ZF and full ZF decreases while user density increases, and is almost zero when $U=A=512$. On the other hand, when the user density is very low, the difference between partial ZF and full ZF is smaller than the difference when the user density is medium because the user spacing is larger and the interference between each user is inherently less in this case.



(a) User-centric



(b) Cluster-centric

Fig. 4. Probability of the number of users considered for weight calculation in each cluster with partial ZF in downlink.

Next, we further analyze the reasons for the largest gap between partial ZF and full ZF in UC and CC approaches. In the ZF scenario, the link capacity degrades in a special case that the number of users considered for weight calculation becomes the same as the number of antennas. This is because the degree of freedom of antennas is all used to multiplex users, and thus, the received signal power for the desired user(s) is reduced. Here, we present the probability of the number of users considered for weight calculation within each cluster during our 100 trials assuming partial ZF in Fig. 4. In the case of CC approach (see Fig. 4 (b)), the probability that the number of users considered for weight calculation in each cluster becomes the same as the number of antennas in each cluster (i.e., $|\mathcal{N}_k| = A_k = 16$) is the highest when $U=64$. Similarly, in the case of UC approach, the probability of $|\mathcal{N}_{u=k}| = A_k = 8$ becomes highest when $U=64$ and 128 (see Fig. 4 (a)). This indicates that the link capacity degradation of partial ZF from full ZF becomes relatively large when $U=64$ and 128 in the UC approach and when $U=64$ in the CC approach.

V. CONCLUSIONS

In this paper, we proposed to incorporate the MU-MIMO technic into UC CF-mMIMO to realize a CC CF-mMIMO. We also derived the general expressions of full ZF and partial

ZF-based pre/postcoding weight vectors by considering the UC approach as a special case of the proposed CC approach. Based on that, the SINR expressions for downlink and uplink were also formulated.

We confirmed by the Monte-Carlo simulation that the proposed CC approach provides a higher capacity and user fairness than the UC approach in the cases of different number of users.

Since the simulation results and corresponding conclusions provided in this paper are based on one particular antenna distribution. How the antenna distribution pattern affects the user capacity and the user fairness is left as our future study. In this paper, we observed that the link capacity degradation of partial ZF from full ZF becomes relatively large when the number of users considered for weight calculation becomes the same as the number of antennas. How to avoid this problem will also be the subject of our future study.

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