# Application of An Efficient Method of Moments to Numerical Analysis of 1-bit Transmitarrays

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*Abstract*—An efficient method of moments for numerical analysis of antennas with variable load impedance is presented. The presented method is a kind of domain decomposition method. A loaded full-matrix equation is decomposed into block matrix equations corresponding to unloaded and loaded parts, respectively. These block matrix equations are solved sequentially using an unloaded full-admittance matrix and currents of the antennas are obtained. The presented method is computationally efficient because matrix inversion of a loaded full-impedance matrix, which is computationally expensive, is unnecessary. The presented method is rigorous because its algorithm includes no approximation. The presented method is applied to numerical analysis of 1-bit transmitarrays and its performance is demonstrated.

*Keywords*—Method of moments, domain decomposition method, transmitarray

#### I. INTRODUCTION

A method of moments (MoM) is well-known as one of the efficient numerical analysis method for antennas[1], [2]. According to recent advancement of antenna technologies, the MoM has been enhanced for numerical analysis of various antennas such as large-scale antennas[3], antennas in layered media[4], and antennas in the vicinity of dielectric bodies [5], [6].

On the other hand, an efficient MoM for numerical analysis of electronically reconfigurable antennas has been expected to be developed. Owing to variable load impedances such as p-i-n diodes, beam scanning capability can be implemented to the electronically reconfigurable antennas [7], [8]. According to such beam scanning capability, it is expected that the electronically reconfigurable antennas are promising technologies for the next generation wireless communication systems. One of the difficulties on design of the electronically reconfigurable antennas is large computational cost because full-wave analysis is necessary every time when their load impedances are tuned.

In this paper, an efficient MoM for numerical analysis of antennas with variable load impedances is presented [9]. The presented MoM is applied for numerical analysis of an one-bit electronically reconfigurable transmitarrays. It is demonstrated that the presented MoM greatly reduce computational cost for designing the one-bit electronically reconfigurable transmitarray.

## II. BRIEF REVIEW OF PRESENTED MOM

According to a geometry of an antenna under test (AUT) and its excitation, a matrix equation to be solved is obtained using the MoM as follows.

$$(\mathbf{Z}_{N \times N} + \mathbf{Z}_{N \times N}^{L})\mathbf{I}_{N} = \mathbf{V}_{N}, \qquad (1)$$

where  $\mathbf{Z}_{N \times N}$  is  $N \times N$  unloaded full-impedance matrix of the AUT,  $\mathbf{Z}_{N \times N}^{L}$  is  $N \times N$  load impedance matrix,  $\mathbf{I}_{N}$ is N dimensional unknown current vector, and  $\mathbf{V}_{N}$  is Ndimensional known voltage vector. It should be noted that  $\mathbf{Z}_{N \times N}^{L}$  is a diagonal matrix and its diagonal entries are nonzero only when load impedance is connected. Eq. (1) can be transformed as follows.

$$\mathbf{I}_N = \mathbf{Y}_{N \times N} (\mathbf{V}_N - \mathbf{Z}_{N \times N}^L \mathbf{I}_N), \qquad (2)$$

where  $\mathbf{Y}_{N \times N} \equiv \mathbf{Z}_{N \times N}^{-1}$  is an unloaded full-admittance matrix.

For numerical analysis of the electronically reconfigurable antenna, Eq. (1) or (2) must be solved every time when the load impedance matrix  $\mathbf{Z}_{N \times N}^{L}$  is tuned. Eq. (1) and (2) are full matrix equations and they are computationally expensive to be solved. On the other hand, as for the electronically reconfigurable antennas, only the load impedance matrix  $\mathbf{Z}_{N \times N}^{L}$  is tuned while the unloaded full-admittance matrix  $\mathbf{Y}_{N \times N}$  is kept. Our presented method enables to solve Eq. (2) efficiently with the help of the unloaded full-admittance matrix  $\mathbf{Y}_{N \times N}$ . Since  $\mathbf{Z}_{N \times N}^{L}$  is a diagonal matrix, Eq. (2) can be expressed using block matrix equations corresponding to loaded and unloaded parts, respectively. Starting from loaded parts of block matrix equations, all block matrix equations including unloaded parts can be solved sequentially once the unloaded full-admittance matrix is obtained in advance [9]. According to limited number of pages, detailed algorithm of the presented MoM is omitted here and interested readers can refer [9].

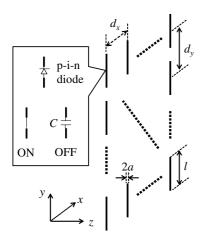


Fig. 1. An 1-bit transmitarray.

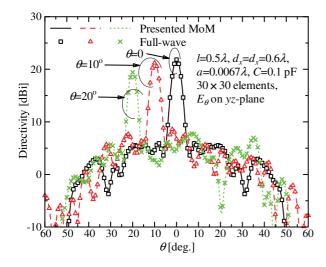


Fig. 2. Directivity of 1-bit transmitarrays.

### III. NUMERICAL ANALYSIS

Numerical analysis model of an 1-bit transmitarray is shown in Fig. 1. Antenna element is a linear dipole element and a p-i-n diode is loaded with it. The p-i-n diode is simply modeled as a short circuit during on-state while it is modeled as a capacitance during off-state. A primary source of the 1bit transmitarray is a Yagi-Uda antenna.

The 1-bit transmitarray is designed so that its main beam is directed to a specific direction. In advance of its design, on/off-states of the p-i-n diodes are switched and phase of transmission coefficient of the isolated elements is obtained. On/off-states of the p-i-n diodes of each dipole element in the designed 1-bit transmitarray are given so that its far field in a specific direction is maximized. Performance of the designed 1-bit transmitarray is obtained using the presented MoM or full-wave analysis. All numerical simulations were performed using Intel Core i5-6300U CPU.

Directivity of the designed 1-bit tranmitarrays is shown in

Fig. 2. It is found that directivity obtained using the presented MoM perfectly agrees with that of full-wave analysis. CPU time for full-wave analysis of these 1-bit tranmitarrays is 2,402 sec. while that for presented MoM is only 738 sec. As mentioned earlier, the presented MoM reuses the unloaded full-admittance matrix. As a result, matrix inversion for the loaded full-matrix equations, which is computationally expensive, is unnecessary every time when the load impedance is tuned. CPU time required for numerical analysis of each configuration of the transmitarrays is just two or three seconds. On the other hand, long CPU time is necessary for full-wave analysis because the loaded full-matrix equations must be solved every time when the load impedance is tuned.

## IV. CONCLUSIONS

An efficient MoM for numerical analysis of antennas with variable load impedance has been presented. The presented MoM has been applied to numerical analysis of the electrically reconfigurable 1-bit transmitarrays. Results of numerical simulations have demonstrated that the presented MoM greatly reduces computational cost.

#### ACKNOWLEDGMENT

We would like to thank staffs in Cyberscience Center, Tohoku University for their helpful advices. This work was financially supported by JSPS KAKENHI Grant Number 18K13736 and 18K04116. We acknowledge the stimulated discussion in the meeting of the Cooperative Research Project of the Research Institute of Electrical Communication,Tohoku University.

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