Evaluation of Q factor of Antennas in Lossy Medium

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***Abstract* - In this paper, the analytical expressions of Q factor of antennas in lossy medium are derived for two models. In model 1, the lossy medium is regarded as a part of the antenna, to emphasize he radiation loss. In model 2, the antenna is located inside a spherically layered lossy medium, to demonstrate the effect of both radiation loss and conductive loss. The analytical expressions are evaluated numerically and the results are compared with the results presented in previous researches.**

***Keywords* — Quality Factor, finite, lossy medium, conductive loss, radiation loss.**

I. INTRODUCTION

The antenna quality factor (Q factor) is a parameter that reflects the antenna's bandwidth and ability on radiation. The antenna Q provides a limitation of antenna performance in antenna design. Generally, the antenna Q is defined as

$$Q = \frac{2\omega W}{P} \tag{1}$$

where W refers to the mean electric energy stored in the space, and P is the power dissipated in radiation.

Studies on antenna Q in free space have been carried out by some researchers before several decades. Different methods have been used in the research. Equivalent circuit theory has been employed in [1]-[2], where P is the power dissipated on the resistance, and W is the energy stored in the inductors and capacitors. In this case, the antenna Q turns out to be an approximate solution. To obtain the exact solution, P and W can be calculated by using the exact electromagnetic fields as in [3]-[5], where P can be obtained from the real part of the Poynting vector, and W can be calculated by subtracting the radiation energy from total energy.

However, in many cases such as the underwater communication and the capsule endoscope, since the operating environment of the antenna is not always loss-free space, the analysis of the Q factor of antennas in the loss medium is desired. In this paper, the analytic expression of antenna Q factor in lossy medium is derived, and the analytic expression is evaluatednumerically.

II. MODEL AND DERIVATION

Figure 1 shows the analytical model. The vertically polarized omni-directional antenna is placed within the sphere space of radius $r=a$. A spherical finite loss medium layer ($a \le r \le b$) with the wave number k_1 is assumed to cover the antenna sphere. The free space is set outside of the radius b with the wave number k_0.

The modal expansion method is used to derive the electromagnetic fields in the lossy medium layer and the outside space. The results are shown in Eq.(2) and Eq.(3).

$$\begin{cases} E_{0\theta} = \dfrac{j\omega\mu_0}{k_0 r} P_n^1(\cos\theta)\left[A_n \hat{H}_n(k_0 r)'\right] \\ E_{0r} = \dfrac{j\omega\mu_0}{k_0^2}\dfrac{n(n+1)}{r^2} P_n(\cos\theta)\left[A_n \hat{H}_n(k_0 r)\right] \quad (r \ge b) \\ H_{0\varphi} = \dfrac{1}{r} P_n^1(\cos\theta)\left[A_n \hat{H}_n(k_0 r)\right] \end{cases} \tag{2}$$

$$\begin{cases} E_{1\theta} = \dfrac{j\omega\mu_0}{k_1 r} P_n^1(\cos\theta)\left[B_n \hat{J}_n(k_1 r)' + C_n \hat{Y}_n(k_1 r)'\right] \\ E_{1r} = \dfrac{j\omega\mu_0}{k_1^2}\dfrac{n(n+1)}{r^2} P_n(\cos\theta)\left[B_n \hat{J}_n(k_1 r) + C_n \hat{Y}_n(k_1 r)\right] \quad (a \le r \le b) \\ H_{1\varphi} = \dfrac{1}{r} P_n^1(\cos\theta)\left[B_n \hat{J}_n(k_1 r) + C_n \hat{Y}_n(k_1 r)\right] \end{cases} \tag{3}$$

A_n, B_n and C_n can be determined by the boundary conditions. By using the conditions on $r=b$, Eq.(4) is obtained. When the antenna model or the source is fixed, all coefficients can be obtained.

$$\begin{aligned} A_n &= \frac{1}{\hat{Y}_n(k_1 b)'\hat{H}_n(k_0 b) - \sqrt{\varepsilon_r}\hat{Y}_n(k_1 b)\hat{H}_n(k_0 b)'} B_n \\ C_n &= \frac{\sqrt{\varepsilon_r}\hat{J}_n(k_1 b)\hat{H}_n(k_0 b)' - \hat{J}_n(k_1 b)'\hat{H}_n(k_0 b)}{\hat{Y}_n(k_1 b)'\hat{H}_n(k_0 b) - \sqrt{\varepsilon_r}\hat{Y}_n(k_1 b)\hat{H}_n(k_0 b)'} B_n \end{aligned} \tag{4}$$

A. *r>b*

In this case, the lossy medium is regarded as a part of the antenna. In the other word, the loss from medium refers to the conductive loss of antenna. Therefore, the antenna Q now represents the radiation Q, which is similar with the one proposed by Chu[1] as Eq.(5). Then A_n, the only coefficient existing, can be influenced by the parameters of lossy medium like permittivity and conductivity.

$$Q = \frac{2\omega W|_{r>b}}{P|_{r=b}} = \frac{\sum_{n=1}^{N} |A_n|^2 \frac{n(n+1)}{2n+1} Q_n(k_0 r)}{\sum_{n=1}^{N} |A_n|^2 \frac{n(n+1)}{2n+1}} \tag{5}$$

B. r>a

In this case, the antenna Q is influenced by both radiation and medium loss. By using the fields in the range a<r<b, P can be derived from the real part of Poynting vector. After subtracting the radiation energy from the sum of total energy in region r>a, the stored energy W can also be obtained. Therefore, the antenna Q in the finite lossy medium can be obtained by the following definition, i.e,

$$Q = \frac{2\omega\left(W|_{a<r<b} + W|_{r>b}\right)}{P|_{r=a}} \quad (6)$$

In order to verify the derivation in this paper, the results are calculated for ε_r=1 and σ=0 by Eq.(5). It can be seen that they agree very well with Collin's results [3].

III. EVALUATION

A. r>b

Fig. 2 shows the radiation Q when ε_r=5 and σ is variable. It is obvious that Q will increase when the mode number N enlarges. When there only exists the primary mode, no matter how the parameters change, all the Q are completely the same, because in this case (N=1), Q is irrelevant to A_n which is influenced by the loss medium parameters. When N=3, it is observed that the radiation Q can be slightly reduced because of the conductive loss, .

B. r>a

Fig. 3 shows the loaded Q for different mode number when ε_r=5 and σ is variable. Similarly, when k_0b becomes bigger than a certain value, it can also be observed that the loaded Q increases with the increase of the mode number N. In addition, it can be seen that the loaded Q decreases obviously when the conductive loss increases, which meets the common sense.

IV. CONCLUSION

The analytical expression for antenna Q in finite lossy medium was derived and the accuracy was evaluated numerically. The results demonstrated the impact on the antenna performance by a conductive loss medium in or around the antennas.

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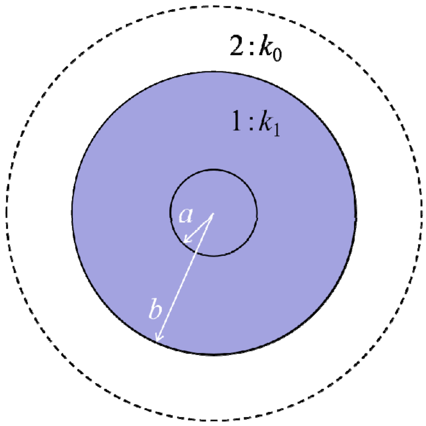

Fig. 1. Two-layer model.

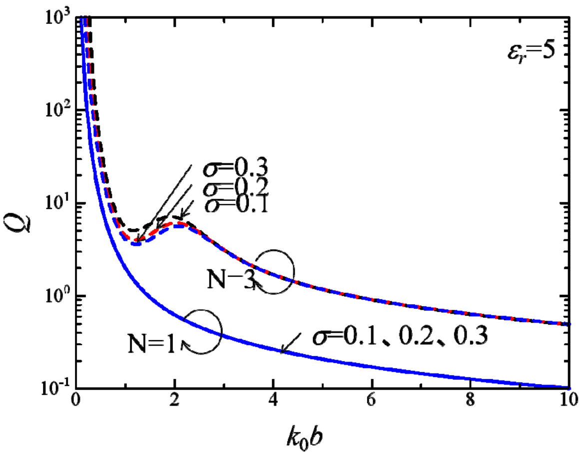

Fig. 2. Quality factor of *r>b* in different mode when ε_r=5 and σ is variable.

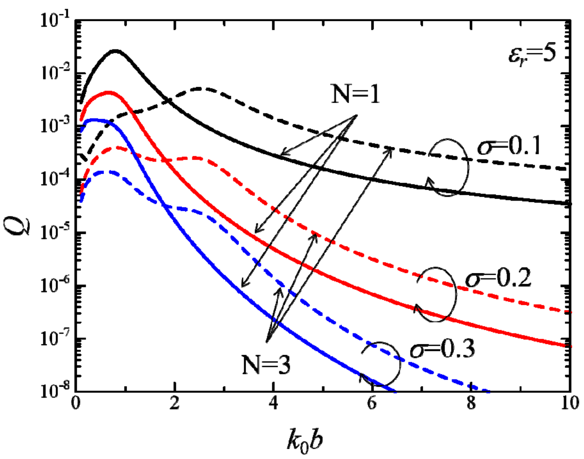

Fig. 3. Quality factor of *r>a* in different mode when ε_r=5 and σ is variable.