

# Design of Finite FSS-backed Reflectarray by Using BDP-CG Method

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**Abstract**—A fast design method of the finite FSS-backed reflectarray is presented. In the design method, induced electromotive force (EMF) method is used for calculating self/mutual impedance between linear elements. Resultant matrix equation is solved by a block diagonal preconditioned-conjugate gradient (BDP-CG) method, not by a conventional direct solver. Because the number of iterations can be saved by the BDP, total CPU time of the BDP-CG method becomes much small compared with that of the conventional direct solver. As a result, total CPU time for the design of reflectarray backed by finite FSS can be saved by using the design method. Numerical example shows the performance of the design method.

## I. INTRODUCTION

A planar reflectarray has received much attention as a candidate of high gain reflector [2]. The planar reflectarray is flat and lightweight while conventional parabolic reflector is bulky and heavy [1]. Another advantage of the planar reflectarray is easy fabrication by using low-cost dielectric substrate.

On the other hand, one of the disadvantages of the planar reflectarray is narrow bandwidth [3]. Therefore, design of broadband/multiband reflectarray has been carried out in previous researches. As a multiband reflectarray, the reflectarray backed by frequency selective surface (FSS) has been proposed [4]-[6]. For the design of FSS-backed reflectarray, numerical analysis of reflectarray element backed by FSS must be carried out repeatedly. In general, the size of FSS is much larger than the size of reflectarray element and numerical analysis of reflectarray element backed by FSS tends to be computationally expensive. Therefore, a fast numerical analysis method for reflectarray element backed by FSS is desired.

For numerical analysis of reflectarray element, finite element method (FEM) with periodic boundary condition (PBC) has been used [7]. Because infinite FSS is assumed in the FEM with PBC, CPU time for numerical analysis of the reflectarray element backed by FSS can be saved. However, incident wave from primary source is approximated as plane wave and edge effect of finite FSS is ignored when the FEM with PBC is used. On the other hand, numerical analysis method of reflectarray element based on an induced electromotive force (EMF) method has been proposed by our group [8]. In our method, self/mutual impedance between linear elements is calculated by the induced EMF method and resultant impedance matrix is

solved by a direct solver such as Gauss-Jordan method. Mutual coupling between primary source and reflectarray element can be calculated exactly by our method while incident wave from primary source is approximated as plane wave in the FEM with PBC. In addition, numerical analysis of reflectarray element backed by FSS can be carried out accurately by our method even when the size of FSS is finite. However, numerical analysis of reflectarray element backed by finite FSS is computationally expensive by using our method which is based on the direct solver.

In this paper, a reflectarray design method which has been proposed in reference [8] is enhanced and applied to the design of FSS-backed reflectarray. In the enhanced method, self/mutual coupling between reflectarray element and FSS element is calculated accurately by using the induced EMF method. Instead of the direct solver, block diagonal preconditioned-conjugate gradient (BDP-CG) method is used in the enhanced method. As a result, CPU time for numerical analysis of reflectarray element backed by finite FSS is greatly reduced. A planar reflectarray which consists of dipole element with/without parasitic dipole element is designed by the enhanced method. Results of numerical analysis show that the accuracy of the enhanced method is practically enough and its CPU time is small.

## II. ENHANCED DESIGN METHOD FOR FSS-BACKED REFLECTARRAY

In this section, the design method for reflectarray which has been proposed in reference [8] is enhanced for the design of FSS-backed reflectarray. In the enhanced method, self/mutual impedance between reflectarray element and FSS element is calculated by an induced electromotive force (EMF) method. Resultant matrix equation is as follows,

$$\begin{bmatrix} \mathbf{Z}_{PP} & \mathbf{Z}_{PF} & \mathbf{Z}_{PR} \\ \mathbf{Z}_{FP} & \mathbf{Z}_{FF} & \mathbf{Z}_{FR} \\ \mathbf{Z}_{RP} & \mathbf{Z}_{RF} & \mathbf{Z}_{RR} \end{bmatrix} \begin{bmatrix} \mathbf{I}_P \\ \mathbf{I}_F \\ \mathbf{I}_R \end{bmatrix} = \begin{bmatrix} \mathbf{V}_P \\ \mathbf{V}_F \\ \mathbf{V}_R \end{bmatrix}, \quad (1)$$

where  $\mathbf{Z}$  is block impedance matrix,  $\mathbf{I}$  is block current vector, and  $\mathbf{V}$  is block voltage vector. Subscript  $P$  means the part of primary source,  $F$  means the part of FSS, and  $R$  means the part of reflectarray element. Instead of a conventional direct solver,

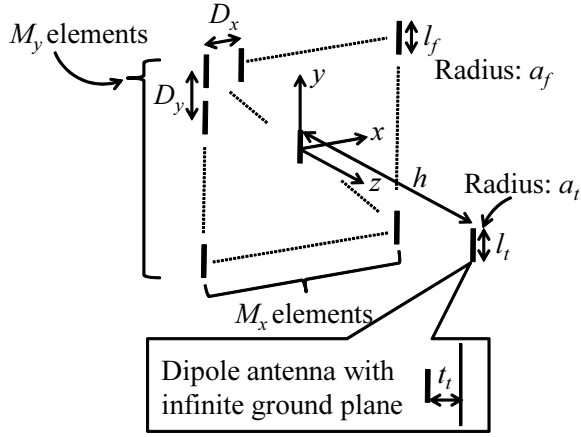


Fig. 1. Bandstop FSS with dipole element.

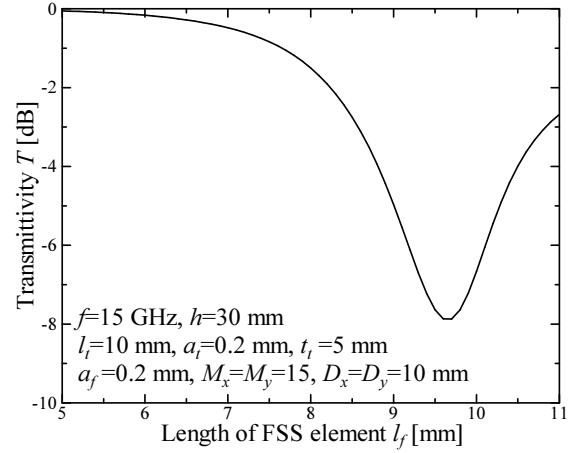


Fig. 2. Transmittivity of bandstop FSS with dipole elements versus length of elements.

Eq. (1) is repeatedly solved by block diagonal preconditioned-conjugate gradient (BDP-CG) method for calculating current of reflectarray element versus its length [9]. Because the part of primary source and FSS in Eq. (1) is large and fixed, the same block diagonal preconditioner corresponding to the part can be used repeatedly to save the number of iterations.

By using the enhanced method, the design of a linear element reflectarray backed by finite FSS is carried out as follows.

- 1) The structure of the reflectarray backed by finite FSS is determined (e.g. number of elements in reflectarray and FSS, array spacing between elements, position of primary source...).
- 2) At each position, current of reflectarray element backed by finite FSS is calculated versus its length. Self/mutual impedance between elements is calculated by using the induced EMF method and resultant matrix equation is solved by the BDP-CG method.
- 3) Phase variation of scattering field to desired  $(\theta_s, \phi_s)$  direction is calculated versus the length of reflectarray element.
- 4) Phase of the total scattering field of FSS to desired  $(\theta_s, \phi_s)$  direction is calculated. The length of reflectarray element at each position is determined as phase of the scattering field of the reflectarray element is identical with that of FSS.

### III. NUMERICAL EXAMPLES

#### A. Design of Bandstop FSS with Dipole Elements

Here, bandstop FSS with dipole elements is designed by using the induced EMF method. In Fig. 1, the bandstop FSS with dipole elements is shown. Primary source is a dipole antenna with infinite ground plane. Transmittivity  $T$  is defined to evaluate the bandstop performance of FSS as follows.

$$T = \frac{\int_0^{2\pi} \int_{\pi/2}^{\pi} |\text{Re}(\mathbf{E} \times \mathbf{H}^*)| \sin \theta d\theta d\phi}{P_{in}/2} \quad (2)$$

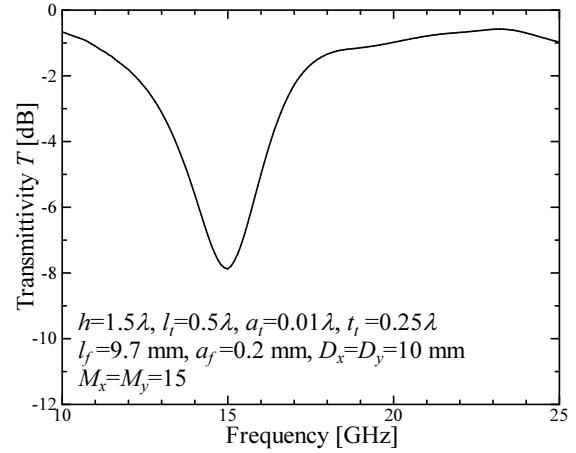


Fig. 3. Frequency characteristic of transmittivity of bandstop FSS with dipole elements.

$\mathbf{E}$  and  $\mathbf{H}$  mean total electric and magnetic field radiated from primary source and FSS, respectively.  $P_{in}$  means the input power of primary source. Numerator of Eq. (2) means total transmitting power to half space of  $z \leq 0$  and transmittivity  $T$  becomes 1(= 0dB) when the FSS does not exist.

A bandstop FSS at 15 GHz is designed. In Fig. 2, transmittivity  $T$  is shown as a function of the length of FSS element. It is found that the bandstop performance of FSS becomes better when the length of element is close to  $\frac{\lambda}{2}$  and the bandstop performance of the FSS becomes maximum at  $l_f = 9.7$  mm. As shown in Fig. 3, the FSS with  $l_f = 9.7$  mm shows bandstop performance at 15 GHz while the FSS shows bandpass performance at the other frequency.

#### B. Design of Linear Element Reflectarray Backed by FSS

As shown in Fig. 4,  $15 \times 15$  linear element reflectarray backed by FSS is designed by the enhanced method. The designed linear element reflectarray consists of dipole element with/without parasitic element. Based on the results of numer-

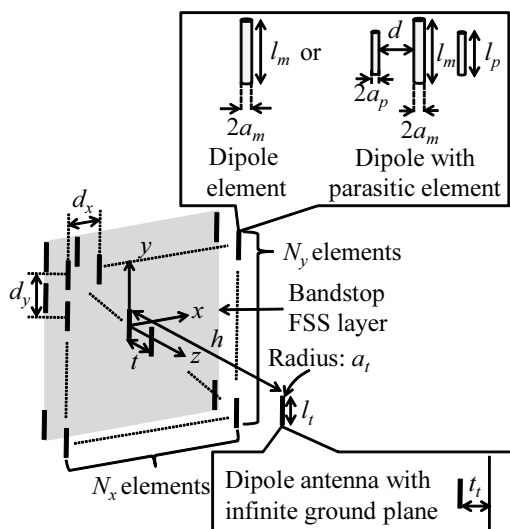


Fig. 4. Linear element reflectarray backed by bandstop FSS.

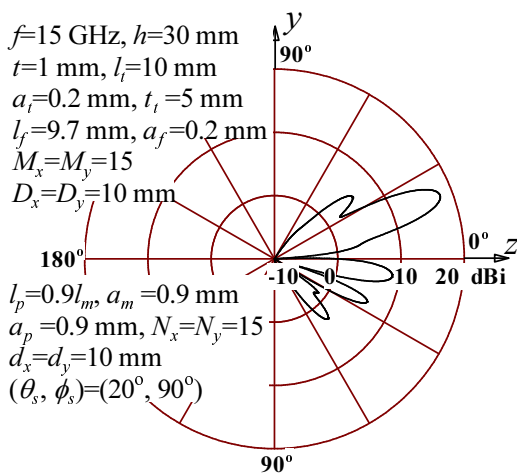


Fig. 5. Scattering pattern of designed linear element reflectarray backed by bandstop FSS.

ical analysis, the length of FSS element  $l_f$  is optimized and  $l_f = 9.7$  mm. In the design example, the main beam direction of the reflectarray is set to  $(\theta_s, \phi_s) = (20^\circ, 90^\circ)$ . Scattering pattern of the designed reflectarray is shown in Fig. 5. The maximum antenna gain is 18 dBi and main beam direction is  $(21.1^\circ, 90^\circ)$ . Because the effect of mutual coupling between reflectarray elements are ignored in the design method, the main beam direction of the designed reflectarray is slightly different from the desired one.

Total CPU time for the design of the  $15 \times 15$  linear element reflectarray backed by FSS was 86,200 sec. when conventional Gauss-Jordan method was used for solving Eq. (1). On the other hand, total CPU time for the design was 8,500 sec. when the BDP-CG method was used for solving Eq. (1). From the results of numerical analysis, it is found that total CPU time for the design of the linear element reflectarray backed

by FSS can be saved by using the BDP-CG method. Eq. (1) is ill-conditioned problem and the number of iterations becomes large when the CG method without the BDP is used for solving Eq. (1). As a result, total CPU time for the design was 113,500 sec. when the CG method without the BDP was used for solving Eq. (1). Therefore, it is found that the BDP is effective technique to save total CPU time of the CG method.

#### IV. CONCLUSIONS

In this paper, the reflectarray design method shown in reference [8] was enhanced for the design of reflectarray backed by finite FSS. The BDP-CG method was introduced to the enhanced method and computational cost for the design of reflectarray backed by finite FSS was reduced drastically. The linear element reflectarray backed by finite FSS with dipole elements was designed by using the enhanced method. Results of numerical analysis showed validity of the enhanced method. The design of multilayer and multiband reflectarray separated by bandstop FSS by the enhanced method and its experimental study is future work.

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