Optimization of Block Size for CBFM in MoM

Keisuke Konno[1], Qiang Chen[1], Kunio Sawaya[1], and Toshihiro Sezai[2]

[1] Department of Electrical and Communications Engineering, Graduate School of Engineering, Tohoku University 6-6-05 Aramaki Aza Aoba, Aoba-ku, Sendai, Miyagi, 980-8579, Japan

[2] Japan Aerospace Exploration Agency, Tsukuba Aerospace Center 2-1-1 Sengen, Tsukuba, Ibaraki, 305-3505, Japan

1 Introduction

Method of Moments(MoM) is known as one of the powerful techniques for numerical analysis of antennas and scatterers[1]. With a direct solver like Gauss-Jordan method, CPU time for the MoM is $O(N^3)$ where N is number of unknowns. Therefore, the direct solver can not be applied to the MoM when N is too large.

Previously, iterative methods like conjugate gradient (CG) method has been proposed for reduction of CPU time [2]. CPU time for each iteration is $O(N^2)$ and total CPU time becomes smaller than $O(N^3)$ when number of iteration steps is smaller than N. However, iterative methods are not effective for ill-conditioned problems because number of iterations for the problems is proportional to N[3]. Therefore, total CPU time required for anlysis of ill-conditioned problems is still $O(N^3)$ even when iterative methods are used.

CBFM (Characteristic Basis Function Method) is also known as one of the powerful techniques for analysis of large-scale problems[4]. Since the CBFM does not include iterative procedure like the CG method, CPU time required for the CBFM can be reduced for the ill-conditioned problem. So far, it has been found that CPU time required for the CBFM depends on number of blocks M. However, relation between number of blocks M and number of segments N, which gives minimum CPU time, has not been investigated. In this paper, optimum number of blocks M is derived theoretically as a function of N. The numerical simulation shows that minimum CPU time for the CBFM is realized by the optimum M.

2 CBFM

2.1 Principle

Unknown N dimensional current vector is expressed as follows,

$$\mathbf{I}_i^{blo} = \sum_{k=1}^{M} \alpha_{(i,k)} \mathbf{J}_{(i,k)}^{blo} \qquad i = 1, 2, ..., M \tag{1}$$

where M is number of blocks, α is unknown weight coefficient, and $\mathbf{J}$ is characteristic basis function (CBF). At first, Primary basis $\mathbf{J}_{(i,i)}^{blo}$ which shows self-interaction on the block are obtained by solving extended matrix equation.

$$\mathbf{Z}_{ii}^{eblo} \mathbf{J}_{(i,i)}^{eblo} = \mathbf{V}_i^{eblo} \qquad i = 1, 2, ..., M, \tag{2}$$

where $\mathbf{Z}_{ii}^{eblo}$ is $(K + K_o) \times (K + K_o)$ extended block matrix, $\mathbf{J}_{(i,i)}^{eblo}$ is $(K + K_o)$ extended block current vector, $\mathbf{V}_i^{eblo}$ is $(K + K_o)$ extended block voltage vector. K is number of segment on the block and K_o is number of overlap segments which improves the CBF by removing undesired edge effects caused by block truncation. Primary basis $\mathbf{J}_{(i,i)}^{blo}$ is obtained from $\mathbf{J}_{(i,i)}^{eblo}$ by excluding values corresponding to overlap segments.

Next, Secondary basis is obtained as follows,

$$\mathbf{Z}_{ii}^{eblo}\mathbf{J}_{(i,k)}^{eblo} = \mathbf{V}_{(i,k)}^{eblo} \quad where \quad \mathbf{V}_{(i,k)}^{eblo} = -\mathbf{Z}_{ik}^{eblo'}\mathbf{J}_{(k,k)}^{blo'} \tag{3}$$
$$(k = 1, 2, ..., i-1, i+1, ..., M)$$

where $\mathbf{Z}_{ik}^{eblo'}$ consists of $(K+K_o) \times K'$ partial matrix of $\mathbf{Z}_{ik}^{eblo}$ and $\mathbf{J}_{(k,k)}^{blo'}$ consists of K' partial vector of $\mathbf{J}_{(k,k)}^{blo}$. $K' = (K - K_o^{ik})$ where K_o^{ik} is number of overlap segments between ith and kth block.

After that, the original matrix equation is transformed into following equation by the CBF.

$$\sum_{i=1}^{M}\sum_{k=1}^{M}\alpha_{(i,k)}\mathbf{u}_{(i,k)} = \mathbf{V} \tag{4}$$
$$(\mathbf{u}_{(i,k)} = [[\mathbf{Z}_{1i}^{blo}\mathbf{J}_{(i,k)}^{blo}][\mathbf{Z}_{2i}^{blo}\mathbf{J}_{(i,k)}^{blo}]\cdots[\mathbf{Z}_{Mi}^{blo}\mathbf{J}_{(i,k)}^{blo}]]^T)$$

Finally, Galarkin procedure is applied to (4) and the original $N \times N$ matrix equation is compressed into $M^2 \times M^2$ reduced matrix. Weighting coefficient α is obtained by solving the reduced matrix and the solution of the original problem can be obtained from (1) .

2.2 Optimum number of blocks

As a function of number of blocks M, order of the CPU time for the CBFM as well as optimum number of blocks M can be shown as follows.

$$\begin{aligned} \text{CPU time} &\propto N^3/M^2 + M^4N \\ &= N^{7/3} \quad \text{where} \quad M \approx 0.9N^{1/3} \end{aligned} \tag{5}$$

The first term of above equation means CPU time for calculation of Primary basis, which shows the most rapid decrease as a function of M. On the other hand, the second term of above equation means CPU time for calculation of reduced matrix, which shows the most rapid increase as a function of M. The optimum number of M can be derived as a value to minimize sum of the two terms. After partial differentiation is applied to Eq.(5), $M \approx 0.9N^{1/3}$ and $O(N^{7/3})$ are easily derived as the optimum M and minimum CPU time, respectively.

3 Numerical Results

In this section, scattering problems are analyzed by the CBFM and the results are compared with those of the CG method as well as Gauss-Jordan method. As shown in Fig. 1, one/two dimensional structures are selected as an analysis model. On the CBFM, number of blocks is selected as around $M \approx 0.9N^{1/3}$ and extended width w_e, which determines number of overlap segments K_0, is also set to be optimum value based on numerical analysis. On the CG method, relative residual $\epsilon = 10^{-4}$ is selected as convergence criteria.

For each antenna, CPU time variation with respect to M is shown in Fig. 2. As previously discussed, it is found that CPU time was minimized when $M \approx 0.9N^{1/3}$. In addition, the result shows that $M \approx 0.9N^{1/3}$ still gives minimum CPU time even when some overlap segments exist.

Total CPU time required for analysis is shown in Fig. 3. CPU time required for the CG method and the CBFM is $O(N^3)$ and $O(N^{7/3})$, respectively. CPU time of the CG method is still $O(N^3)$ since linear antenna is an ill-conditioned problem and number of iteration steps is proportional to N. On the other hand, CPU time required for the CBFM is $O(N^{7/3})$ because the CBFM does not include iterative procedure and can realize constant CPU time for any problems. From the results, it is shown that the CBFM is more effective for ill-conditioned problems than the CG method.

4 Conclusion

Relation between M and N which realizes minimum CPU time of the CBFM was theoretically derived and $M = 0.9N^{1/3}$. After that, numerical analysis of one/two dimensional antenna structure was carried out by using the CBFM. On the numerical analysis, it was found that minimum CPU time required for analysis by the CBFM is $O(N^{7/3})$ when $M = 0.9N^{1/3}$.

5 Acknowledgements

This work was supported by the GCOE Program CERIES in Tohoku University, and SCAT (Support Center for Advanced Telecommunications) Technology Research, Foundation. Supercomputing resources at Cyberscience Center in Tohoku University were used in this research.

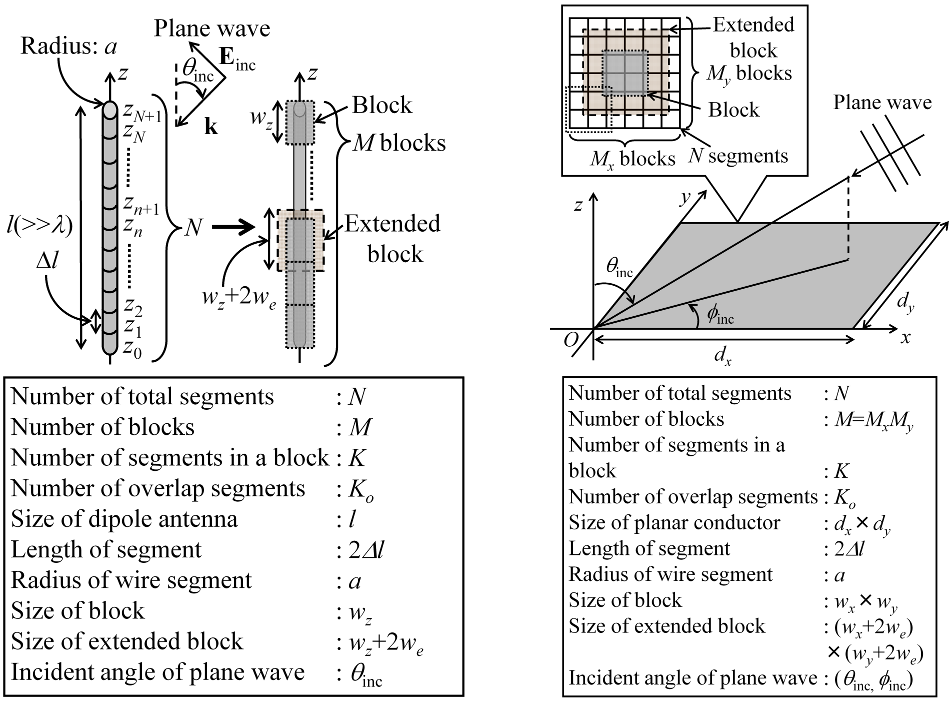
References

[1]R.F. Harrington, Field Computation by Moment Methods, Macmillan, New York, 1968.

[2]R. Coifuman, V. Rokhlin, and S. Wandzura, “The fast multipole method for the wave equation: A pedestrian prescription,” IEEE Antennas and Propag. Mag., vol.35, no.3, pp.7-12, June 1993.

[3]K. Konno, Q. Chen, and K. Sawaya, “Quantitative evaluation for computational cost of CG-FMM on typical wiregrid models,” IEICE Trans. Commun., vol.E93-B, no.10, pp.2611-2618, Oct. 2010.

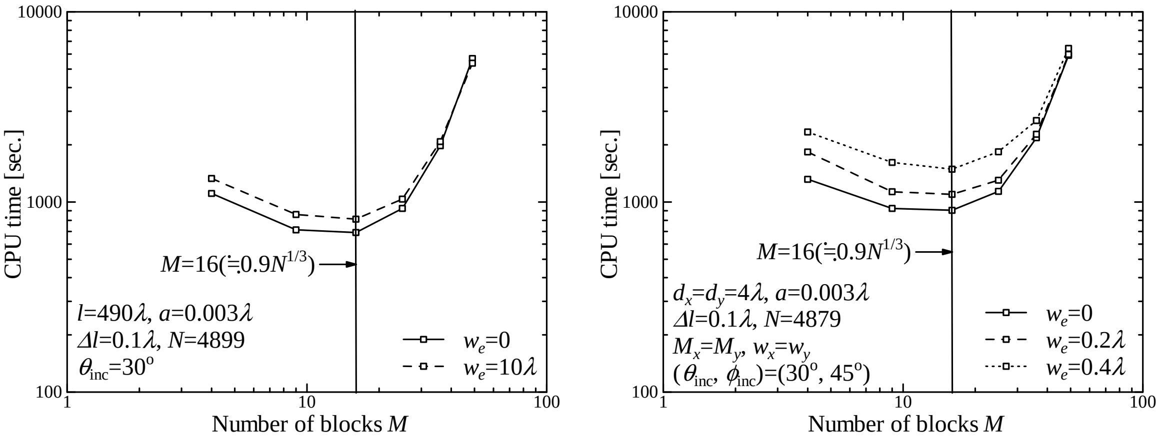
[4]V.V.S. Prakash and R. Mittra, “Characteristic basis function method: A new technique for efficient solution of method of moments matrix equations,” Microw. Opt. Technol. Lett., vol.36, no.2, pp.95-100, Janu. 2003.



(a) Linear antenna.

(b) Planar antenna.

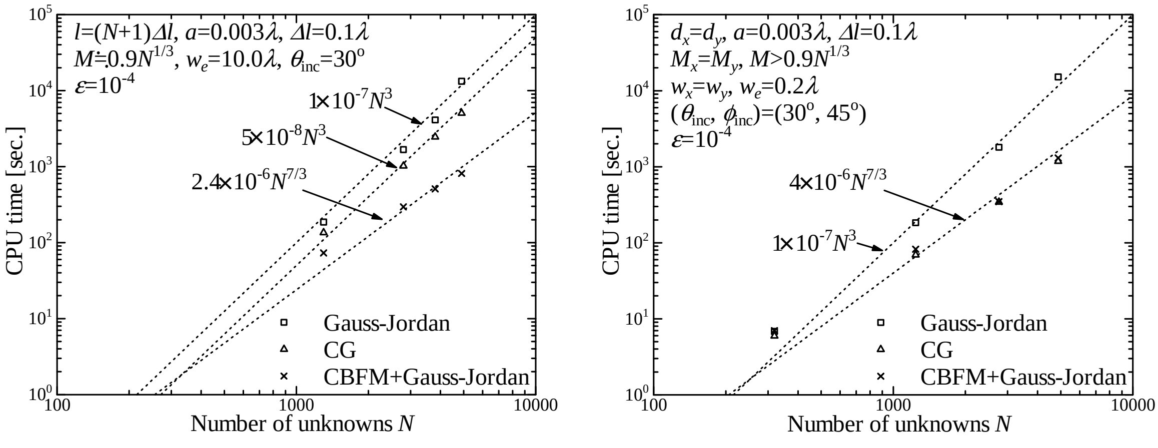
Figure 1: Analysis model.



(a) Linear antenna.

(b) Planar antenna.

Figure 2: CPU time variation with respect to M.



(a) Linear antenna.

(b) Planar antenna.

Figure 3: Total CPU time.