Simulation of Electromagnetic Transients of the Bus Bar in Substation by the Time-Domain Finite-Element Method

Lei Liu, Xiang Cui, Senior Member, IEEE, and Lei Qi

Abstract—In order to analyze the electromagnetic interference (EMI) generated from the aerial bus bars in substation, the transient electromagnetic wave process on the bus bars is calculated by using the time-domain finite-element (TDFE) method. The TDFE method is preferable to both the commonly used finite-difference time-domain (FDTD) method, which has difficulties in dealing with the multiconductor transmission lines (MTLs) with lumped parameter networks, and the universal software electromagnetic transient program (EMTP), which is not effective for the calculation of the whole electromagnetic wave processes along the MTLs. The feasibility and efficiency of the proposed TDFE method have been demonstrated by comparing the numerical results with experimental measurements. Furthermore, we have performed a successful case study on the numerical prediction of the EMI in the secondary cable in substation by using the TDFE method.

Index Terms—Capacitance voltage transformer (CVT), lumped parameter network, multiconductor transmission lines (MTLs), time-domain finite element (TDFE).

I. INTRODUCTION

THE INTENSE transient electromagnetic (EM) wave processes along bus bars and power lines can be caused by lightning or the fault and switching operations in substation. The frequency spectrum of the transient wave can reach 10 MHz. These transient voltages will be coupled in secondary circuits via capacitive voltage transformer (CVT) [1] and become an EM interference (EMI) for the electronic devices and equipment in the substation. Therefore, we should thoroughly understand these transient wave processes in order to forecast possible malfunction in the substation. In order to predict the transient wave process, the bus bars and power lines are generally modeled as multiconductor transmission lines (MTLs) [2]–[4].

In general, the transient simulation of the MTLs can be performed by the finite-difference time-domain (FDTD) method [5], [6]. As well known, FDTD is based on either calculation of electric and magnetic fields [7], which has been applied to very fast surge simulation [8]–[10], or calculation of the parameters, voltages, and currents, which is referred in this paper. However, it is difficult to use the FDTD method to simulate the MTLs with lumped parameters. At first, the state equations of lumped parameter network should be solved [11]. Second, the FDTD method will lead to the unstable calculating result and Gibbs phenomenon caused by the central difference. In addition, the universal software EMTP, which is based on Bergeron’s method, can only be suitable for the calculation of the voltage and current at some specified nodes, and it is not effective to calculate the whole EM wave processes along the MTLs [12], [13].

It is well known that the finite-element method (FEM) has been widely used for solving various EM problems [14]–[21]; however, this powerful method has rarely been applied to the analysis of the transient wave process of the MTLs. In this paper, instead of the FDTD method, we use the time-domain finite-element (TDFE) method to calculate the transient wave process of the MTLs. At first, the arbitrary finite element of the MTLs is integrated, respectively, in time and spatial domains, and we can obtain the local finite-element model. Second, the global finite-element model can be obtained by assembling all the local finite elements together. The MTL with lumped parameter network can be simply solved by the aforementioned approach. Finally, as an example, by use of the TDFE method, we calculate the interference voltage in a secondary cable in a substation.

II. METHODOLOGY

A. Local Finite-Element Model

When we solve the telegrapher’s equations by use of the TDFE method, the MTLs need to be subdivided into a finite number of elements. Each node of the element has a local coordinate and a global coordinate. Assuming that the global system is divided into \(M\) finite elements with \(S\) nodes, Fig. 1 shows one finite-element model of the \(N\)-conductor lines.

![Finite-element model of the \(N\)-conductor lines.](image)
In Fig. 1, \( U_{1,1}, \ldots, U_{1,N} \) and \( I_{1,1}, \ldots, I_{1,N} \) denote the voltages and currents corresponding to local node “1”; similarly, \( U_{2,1}, \ldots, U_{2,N} \) and \( I_{2,1}, \ldots, I_{2,N} \) denote the voltages and currents corresponding to local node “2.” \( R, L, G, \) and \( C \) denote the per-unit-length (p.u.l.) resistance, inductance, conductance, and capacitance matrices of the MTLs, and each of them is an \( N \times N \) matrix. From the telegrapher’s equations, we can get

\[
\frac{\partial U(z,t)}{\partial z} + L \frac{\partial I(z,t)}{\partial t} + RI(z,t) = 0
\]

(1)

\[
\frac{\partial I(z,t)}{\partial z} + C \frac{\partial U(z,t)}{\partial t} + GU(z,t) = 0
\]

(2)

where \( z \) is the propagating direction, \( U \) and \( I \) are the column vectors of the voltage and current at point \( z \) and time \( t \). Using the weighted residue method, we have

\[
\int_{z_1}^{z_2} \left[ \frac{\partial U(z,t)}{\partial z} + L \frac{\partial I(z,t)}{\partial t} + RI(z,t) \right] W_1 \, dz = 0
\]

(3)

\[
\int_{z_1}^{z_2} \left[ \frac{\partial U(z,t)}{\partial z} + C \frac{\partial U(z,t)}{\partial t} + GU(z,t) \right] W_2 \, dz = 0
\]

(4)

where \( z_1 \) and \( z_2 \) are the coordinates of nodes 1 and 2, and \( W_1, W_2 \) are the \( 1 \times N \) weighting function matrices. The voltage and current in every finite element can be approximated as a linear superposition of interpolation functions, i.e.,

\[
U(z,t) = \sum_{k=1}^{2} \Phi_k(z) U_k(t)
\]

(5)

\[
I(z,t) = \sum_{k=1}^{2} \Phi_k(z) I_k(t)
\]

(6)

where \( U_k(t) \) and \( I_k(t) \) are the column vectors of the voltage and current at local node \( k \) \((k = 1, 2)\), and can be represented as

\[
U_k(t) = [U_{k,1}(t), U_{k,2}(t), \ldots, U_{k,N}(t)]^T
\]

(7)

\[
I_k(t) = [I_{k,1}(t), I_{k,2}(t), \ldots, I_{k,N}(t)]^T
\]

(8)

and the two interpolation function matrices \( \Phi_k(z)(k = 1, 2) \) are, respectively, represented as

\[
\Phi_1(z) = \text{diag}\left( \frac{z_2 - z}{z_2 - z_1}, \frac{z_2 - z}{z_2 - z}, \ldots, \frac{z_2 - z}{z_2 - z_1} \right)
\]

(9)

\[
\Phi_2(z) = \text{diag}\left( \frac{z - z_1}{z_2 - z_1}, \frac{z - z_1}{z_1 - z_1}, \ldots, \frac{z - z_1}{z_2 - z_1} \right)
\]

(10)

Substituting (5)–(10) for \( U \) and \( I \) in (3) and (4) and letting \( W_1 = W_2 = \text{diag}(1, 1, \ldots) \), we have

\[
L \frac{\Delta z}{2} \sum_{k=1}^{2} \frac{dI_k(t)}{dt} + R \frac{\Delta z}{2} \sum_{k=1}^{2} I_k(t) + U_2(t) - U_1(t) = 0
\]

(11)

\[
C \frac{\Delta z}{2} \sum_{k=1}^{2} \frac{dU_k(t)}{dt} + G \frac{\Delta z}{2} \sum_{k=1}^{2} U_k(t) + I_2(t) - I_1(t) = 0
\]

(12)

where \( \Delta z = z_2 - z_1 \) is the length of the finite element. Calculating the time integral of (11) and (12), we can get

\[
\int_{t_1}^{t_2} \left[ L \frac{\Delta z}{2} \sum_{k=1}^{2} \frac{dI_k(t)}{dt} + R \frac{\Delta z}{2} \sum_{k=1}^{2} I_k(t) + U_2(t) - U_1(t) \right] dt = 0
\]

(13)

and then, we have

\[
\begin{align*}
L \frac{\Delta z}{2} (\Delta I_1 + \Delta I_2) + R \frac{\Delta z}{2} \left[ \frac{1}{2}(\Delta I_1 + \Delta I_2) + (I_1^e + I_2^e) \right] \\
+ \frac{1}{2} (\Delta U_2 - \Delta U_1) \Delta t + (U_2^n - U_1^n) \Delta t = 0 \\
C \frac{\Delta z}{2} (\Delta U_1 + \Delta U_2) + G \frac{\Delta z}{2} \left[ \frac{1}{2} (\Delta U_1 + \Delta U_2) + (U_1^n + U_2^n) \right] \\
+ \frac{1}{2} (\Delta I_2 - \Delta I_1) \Delta t + (I_2^n - I_1^n) \Delta t = 0.
\end{align*}
\]

(14)

Transforming (14) into the form of matrix, we can get

\[
\begin{bmatrix}
-\frac{1}{2} \Delta t & \frac{1}{2} \Delta t \\
\frac{1}{2} \Delta z (C + \frac{1}{2} \Delta t G) & \frac{1}{2} \Delta z \left( C + \frac{1}{2} \Delta t G \right)
\end{bmatrix}
\begin{bmatrix}
\Delta U_1 \\
\Delta U_2
\end{bmatrix}
+ \begin{bmatrix}
\frac{1}{2} \Delta z \left( L + \frac{1}{2} \Delta t R \right) & \frac{1}{2} \Delta z \left( L + \frac{1}{2} \Delta t R \right)
\end{bmatrix}
\begin{bmatrix}
\Delta I_1 \\
\Delta I_2
\end{bmatrix}
= \begin{bmatrix}
-\Psi_1 \\
-\Psi_2
\end{bmatrix}
\]

(15)

\[
\Psi_1 = \frac{1}{2} \Delta z \Delta t (I_1^n + I_2^n) + \Delta t (U_2^n - U_1^n)
\]

(16)

\[
\Psi_2 = \frac{1}{2} \Delta z \Delta t (U_1^n + U_2^n) + \Delta t (I_2^n - I_1^n)
\]

(17)

where

\[
\Delta U_k = U_k^{n+1} - U_k^n \quad (k = 1, 2)
\]

(18)

\[
\Delta I_k = I_k^{n+1} - I_k^n \quad (k = 1, 2)
\]

(19)

\( \Delta t = \text{diag} \{ \Delta t, \Delta t, \ldots, \Delta t \} \), with \( \Delta t \) being the time step, the superscript denotes time series, and \( n \) is the natural number.

In order to satisfy the Kirchhoff’s current law for each node in the global system, it is necessary to transform (15) into

\[
\begin{bmatrix}
\Delta I_1 \\
\Delta I_2
\end{bmatrix}
= A \begin{bmatrix}
\Delta U_1 \\
\Delta U_2
\end{bmatrix}
+ B \begin{bmatrix}
-\Psi_1 \\
-\Psi_2
\end{bmatrix}
\]

(20)

where \( A \) is the element stiffness matrix and \( B \) is the coefficient matrix.
B. Situation of Frequency Dependence

Considering the frequency dependence of the MTLs and according to the time-domain convolution algorithm, we can obtain the recursion formulas of the voltage and current at the local node.

In general, the frequency-dependent parameters of the MTLs are obtained from a series of discrete frequencies within the interested bandwidth. Thus, we should first solve the equations in frequency domain, and then turn it into the time domain by applying inverse Fourier transform [22]–[24]. So, the impedance (Z) and admittance (Y) matrices p.u.l. should be calculated by using a vector-fitting algorithm [25]–[27], i.e.,

\[ Z \equiv \left( L + \sum_{m=1}^{N_s} b_m \frac{s}{s - a_m} \right) s = 0 \]  

\[ Y \equiv \left( C + \sum_{m=1}^{N_s} q_m \frac{s}{s - p_m} \right) s = 0 \]  

where \( L \) and \( C \) are the inductance and capacitance constant matrices p.u.l., \( b_m \) and \( q_m \) are residue matrices, \( a_m \) and \( p_m \) are poles, and \( N_s \) is the number of the poles. Equations (21) and (22) in the time domain can be represented as the sum of the corresponding finite term exponential functions, and these equations in the time domain are convenient for time convolution. Substituting (21) and (22) into (1) and (2) and by the use of inverse Laplace transform, we can obtain the following:

\[ \frac{\partial U(z, t)}{\partial z} + L \frac{\partial I(z, t)}{\partial t} + \sum_{m=1}^{N_s} b_m e^{a_m t} \ast \frac{\partial I(z, t)}{\partial t} = 0 \]  

\[ \frac{\partial I(z, t)}{\partial z} + C \frac{\partial U(z, t)}{\partial t} + \sum_{m=1}^{N_s} q_m e^{p_m t} \ast \frac{\partial U(z, t)}{\partial t} = 0 \]  

where \( \ast \) denotes time convolution.

Using the same procedure as that in Section II-A, (23) and (24) become

\[ \begin{bmatrix} \frac{1}{2} \Delta t & \frac{1}{2} \Delta t \\ \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \Delta t \mathbf{E}_1 & \frac{1}{2} \Delta t \mathbf{E}_2 \\ \end{bmatrix} \begin{bmatrix} \Delta U_1 \\ \Delta U_2 \end{bmatrix} \]  

\[ \begin{bmatrix} \frac{1}{2} \Delta t & \frac{1}{2} \Delta t \\ \end{bmatrix} \begin{bmatrix} \Delta I_1 \\ \Delta I_2 \end{bmatrix} = \begin{bmatrix} -\Psi_1 \\ -\Psi_2 \end{bmatrix} \]  

where

\[ \begin{align*} 
\mathbf{E}_1 &= \sum_{m=1}^{N_s} p_m \left( -\frac{1}{q_m} - \frac{1}{\Delta t q_m^2} (1 - e^{q_m \Delta t}) \right) \\
\mathbf{E}_2 &= \sum_{m=1}^{N_s} b_m \left( -\frac{1}{a_m} - \frac{1}{\Delta t a_m^2} (1 - e^{a_m \Delta t}) \right) \\
\Psi_1 &= \Delta t (U_2^n - U_1^n) + \varphi_1^n \\
\end{align*} \]  

Equation (25) should also be transformed into the form as (20).

C. Global Finite-Element Model

In MTLs, each node has its local and global coordinates, as shown in Fig. 2. In order to get the global stiffness matrix, every node of the system should satisfy the following equation:

\[ \int_{t_1}^{t_2} \sum_{j=1}^{m} (-1)^j \mathbf{I}_{j,p} \cdot (\mathbf{I}_F)_s \, dt = 0 \]  

where \( s = 1, 2, \ldots, NDZ \) is the node coordinate in the global system, \( NDZ \) is the total number of nodes, \( m \) is the total number of the elements around node \( s \), \( p = 1, 2 \) denotes the local node coordinate of the element \( j \), which is connected to node \( s \), and \( \mathbf{I}_F \) is the injected current in global node \( s \). Calculating the trapezoidal integral of (30), we obtain

\[ F_s + \sum_{j=1}^{m} \frac{\partial F_s}{\partial (I_{j,p})_s} \left[ (\Delta I_{j,p})_s \right] = 0 \]  

where

\[ \begin{align*} 
F_s &= \sum_{j=1}^{m} (-1)^j \frac{1}{2} \left[ (\mathbf{I}_{j,p})_s + (\mathbf{I}_{j,p}^{n+1})_s \right] \Delta t \\
&\quad + \frac{1}{2} \left[ (\mathbf{I}_F)_s + (\mathbf{I}_F^{n+1})_s \right] \Delta t. 
\end{align*} \]
According to (20), the increment of current of each finite element can be written as
\[
\Delta I_1 = -A_{11} \Delta U_1 - A_{12} \Delta U_2 - B_{11} \Psi_1 - B_{12} \Psi_2 
\]
\[
\Delta I_2 = -A_{21} \Delta U_1 - A_{22} \Delta U_2 - B_{21} \Psi_1 - B_{22} \Psi_2. 
\]

In the global system, for an arbitrary node \( s \), if the local coordinate in the element \( j \) is equal to 1, then we have
\[
[A_{11} \quad A_{12}] \begin{bmatrix} \Delta U_1 \\ \Delta U_2 \end{bmatrix} = -(B_{11} \Psi_1 + B_{12} \Psi_2) + 2 \\
\quad \cdot (I_f^s_1 - [(I_f^s_1)^2] + (I_f^s_1)^2). 
\]

If the local coordinate in the element \( j \) is equal to 2, then we have
\[
- [A_{21} \quad A_{22}] \begin{bmatrix} \Delta U_1 \\ \Delta U_2 \end{bmatrix} = (B_{21} \Psi_1 + B_{22} \Psi_2) - 2 \\
\quad \cdot (I_f^s_2 - [(I_f^s_1)^2] + (I_f^s_1)^2). 
\]

According to the Kirchhoff’s current law, we can obtain the iterative matrix for the global system, i.e.,
\[
A_g [\Delta U_g] = b_g \tag{37} 
\]
where the global stiffness matrix \( A_g \) is symmetric, and each time step \( b_g \) can be obtained from the former time step.

To ensure the convergence of the aforementioned algorithm, it is required that the propagating distance of the transient wave must be less than \( \Delta z \) within one time step. We use \( \Delta t = \Delta z/\nu \) in this paper, where \( \nu \) is the maximum mode velocity of electromagnetic wave and can be known from the mode analysis of the MTLs.

**D. TDFE Method for Lumped Parameter**

For the node network with lumped parameters, we have to solve the state equation of the network if we use the FDTD method. If every lumped parameter of the network is modeled as a finite element, the MTLs with lumped parameter can be easily analyzed.

The equations for the resistance and inductance can be directly obtained from (15). As for the capacitance shown in Fig. 3, according to its voltage-to-current correlation \( c \Delta u(t)/dt = i(t) \), we can obtain the following equations for local nodes 1 and 2:
\[
c \left\{ \left( u_1^{n+1} - u_2^{n+1} \right) - \left( u_1^n - u_2^n \right) \right\} = \frac{1}{2} \left( i_1^{n+1} + i_1^n \right) \Delta t 
\]
\[
c \left\{ \left( u_2^{n+1} - u_1^{n+1} \right) - \left( u_2^n - u_1^n \right) \right\} = -\frac{1}{2} \left( i_2^{n+1} + i_2^n \right) \Delta t. 
\]

From (38) and (39), we can obtain the finite-element equation of the capacitance
\[
\begin{bmatrix} C_1 & -C_1 \\ -C_1 & C_1 \end{bmatrix} \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \end{bmatrix} + \begin{bmatrix} -\frac{1}{2} \Delta t & 0 \\ 0 & \frac{1}{2} \Delta t \end{bmatrix} \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} = 0 \tag{40} 
\]
\[
\Psi_1 = -i_1^n \Delta t \tag{41} \\
\Psi_2 = i_2^n \Delta t. \tag{42} 
\]

**III. Verification**

**Example 1:** This example consists of a 0.5-m-long coupled line. The input signal is unit step voltage. The coupled lines are terminated at near ends and far ends all with 50 \( \Omega \) resistors (see Fig. 4).

The p.u.l. capacitance matrix and the inductance matrix are as follows:
\[
L = \begin{bmatrix} 1.07754 & 0.538771 \\ 0.538771 & 0.805756 \end{bmatrix} \times 10^{-6} \text{ H/m} 
\]
\[
C = \begin{bmatrix} 71.8544 & -58.8956 \\ -58.8956 & 117.791 \end{bmatrix} \times 10^{-12} \text{ F/m}. 
\]

The transient voltages at the terminals of the coupled lines are shown in Fig. 5, respectively, along with the results obtained using the FDTD method and TDFE method. It is shown that the results match well with each other. But in the calculated results by use of the FDTD method, the waveforms appear some oscillation (as seen in Fig. 5). These oscillations are Gibbs phenomenon caused by the central difference, which is used in the process of calculation. Although the TDFE method is integral calculation, the voltage and the current of the discrete points are in the same location, so that we can better suppress these oscillations.

On the other hand, the FDTD method has stringent requirements for the spatial step \( \Delta z \), and different \( \Delta z \) corresponds to a larger difference between calculated results. The spatial grid points of the interconnect in the FDTD method is sampled with \( N = 20, 10, 5 \), and the calculated results are shown in Fig. 6(a). Fig. 6(b) shows the calculated results of the TDFE method, with the spatial step chosen equal to the FDTD method. It is clear that the calculation error has shown a corresponding increase with the spatial step enlarged. While in the method used in this paper, the calculated results are much more stable with the different spatial step. Using this advantage of the TDFE method, we can solve the state equation of the network if we use the FDTD method. If every lumped parameter of the network is modeled as a finite element, the equations for the resistance and inductance can be directly obtained from (15).
Fig. 5. Transient voltages at two terminations of the coupled lines. (a) Transient response at A and B. (b) Transient response at C and D.

Fig. 6. Calculated results of the TDFE method and FDTD method when the coupled lines are dispersed into different spatial step. (a) Calculated results of the FDTD method. (b) Calculated results of the TDFE method.

method, it could save vast amounts of computing resource when the computing network scale is large.

Example 2: In this example, an experiment has been done to verify the feasibility of the TDFE method. Considering a coaxial cable (RG58A/U) with two branches opening on the terminal, as shown in Fig. 7, the sectional area of the cable core wire is 16.4 mm². The shielding layer of the cable is taken as the reference conductor. Choosing one cable with 10 m in length, we measured the frequency-dependent parameters p.u.l. of the cable by use of the method described in [28] and [29] with Angilent network analyzer (Model 4395A). The experimental results are shown in Fig. 8. It is obvious that these parameters are frequency dependent, and this frequency dependence must be considered for the calculation of the voltage response at the four points 1, 2, 3, 4, as shown in Fig. 4.

The waveform of the input voltage generator (Model HP 33120A), as shown in Fig. 7, is shown in Fig. 9. The transient voltage at points 1, 2, 3, 4 in Fig. 7 was measured by use of a digital storage oscilloscope (Model TDS2024). The recorded waveforms of these voltage responses are shown as red lines in Fig. 10. We have also calculated the corresponding voltage responses by use of the proposed TDFE method, and the simulated waveforms are shown as blue lines in Fig. 10.

By comparing red lines with blue lines in Fig. 10, we can see that the simulated waveforms have good agreement with the measured waveforms. These experimental results demonstrate the feasibility and efficiency of the proposed TDFE method.

IV. APPLICATION

The switching operation in substation will cause transient voltage pulses with high frequency along the bus bars. This kind of voltage pulse will result in the EMI at the terminal of secondary cables through the connection of the CVT and the bus bars, as shown in Fig. 8. Because it is complicated for the FDTD method to analyze the MTLs with lumped parameters, only the transient voltages along the bus bars are calculated in [31], and use these voltage responses as exciting sources to predict the secondary interference voltage of the CVT. But the authors ignored the reflection between the bus bars and the CVT. In fact, the transient voltage caused by switching operation will lead to an interference electric potential (IEP), because of the existences of the inductance of ground leading wire and the impedance of earth mesh. This IEP will be coupled to the secondary system of the CVT through the faraday shielding capacitance. The disturbance voltage cause by IEP at the terminal of the secondary cable was predicted by using the TDFE method in this paper.
As the frequency is increased, the current over the wire cross section tends to crowd closer to the outer periphery because of a phenomenon known as skin effect. Essentially, the current can be assumed to be concentrated in an annulus at the wire surface of thickness \( \delta = \frac{1}{\pi} \sqrt{\frac{\mu_0 \sigma f}{\pi}} \), equal to the skin depth when the skin depth is less than the wire radius. Since the resistance is proportional to the cross-sectional area occupied by the current and the skin depth \( \delta \) decreases with increasing frequency as the inverse square root of frequency, the p.u.l. resistance becomes larger with frequency increasing.

The internal inductance of the wire is due to magnetic flux internal to the wire. For high-frequency excitation, the current again tends to crowd the wire surface and tends to be concentrated in a thickness \( \delta \). The p.u.l. internal inductance for these higher frequencies is derived in [30] and becomes

\[
L = \left( \frac{2}{\pi} \right) \left( \frac{\mu_0}{8\pi} \right) \left( \frac{1}{\sqrt{f}} \right) (r) \quad \sigma \quad \text{(r is the radius of the wire and f is the frequency)}.
\]

So, the p.u.l. internal inductance decreases with the increasing frequency.

Increasing frequency has little effect on the capacitor. The change could be ignored. (c) Conductance is related to the dielectric loss. Since the dielectric loss will be increased at higher frequency, so does the conductance.
In Fig. 11, the CVT is connected to the aerial bus bars. $L_G$ is the inductance of the ground leading wire, $C_3$ represents the Faraday shielding capacitance between the secondary winding of the CVT and the ground leading wire, $C_1$ and $C_2$ are the CVT capacitances, and $Z'$ is the damped impedance of the primary side of the CVT. $C_1$ and $C_2$ are shorted during switching operation. A considerable transient voltage pulse will occur at the secondary cable through the Faraday shield capacitance. $Z'$ acts as an open circuit at high frequency.

The equivalent circuit of the CVT, as shown in Fig. 11, is shown in Fig. 12. The bus bars are 16 m in height, 8 m in space interval, and 90 m in length. A, B, and C are the 500-kV three-phase voltage sources with the industrial frequency of 50 Hz. $C_1 = 1670 \text{ pF}$, $C_2 = 8000 \text{ pF}$, $L_G = 0.1 \text{ } \mu \text{H}$, and $C_3 = 500 \text{ pF}$. The resistance of ground grid is $Z_G = 0.2 \text{ } \Omega$.

The secondary cable is the ZR-VV22-5*4 polyvinyl chloride insulation shield installation wire heatproof control cable. By using of the image method, the p.u.l. parameter of the cable can be calculated [11]. The p.u.l. parameters of the cable can be obtained as follows:

$$
C = \begin{bmatrix}
0.2678 & -0.0491 & -0.0081 & -0.0081 & -0.0491 \\
-0.0491 & 0.2678 & -0.0491 & -0.0081 & -0.0081 \\
-0.0081 & -0.0491 & 0.2678 & -0.0491 & -0.0081 \\
-0.0491 & -0.0081 & -0.0491 & 0.2678 & -0.0491 \\
-0.0491 & -0.0081 & -0.0081 & -0.0491 & 0.2678 \\
\end{bmatrix} \times 10^{-9} \text{ C/m.}
$$

Conductance is very small and can be ignored. The resistance matrix $R$ is obtained from the direct current resistance of the cable

$$
R = \begin{bmatrix}
0.00461 & 0 & 0 & 0 & 0 \\
0 & 0.00461 & 0 & 0 & 0 \\
0 & 0 & 0.00461 & 0 & 0 \\
0 & 0 & 0 & 0.00461 & 0 \\
0 & 0 & 0 & 0 & 0.00461 \\
\end{bmatrix} \text{ } \Omega/\text{m.}
$$

The terminal resistance is $Z_L = 1000 \text{ } \Omega$. Using the proposed method, we calculated the interference voltage and current at the end (point D) of secondary cable of the CVT. Table I shows the peak-to-peak voltages and currents at point D for different preliminary phases.

<table>
<thead>
<tr>
<th>Preliminary Phases</th>
<th>$\theta$</th>
<th>$\pi/6$</th>
<th>$\pi/3$</th>
<th>$\pi/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_L$ (kV)</td>
<td>10m</td>
<td>27.47</td>
<td>23.95</td>
<td>14.02</td>
</tr>
<tr>
<td>50m</td>
<td>25.2</td>
<td>21.97</td>
<td>12.86</td>
<td>0.557</td>
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<td>75m</td>
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<td>21.89</td>
<td>12.81</td>
<td>0.4385</td>
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<td>100m</td>
<td>25.1</td>
<td>21.89</td>
<td>12.81</td>
<td>0.4385</td>
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Fig. 13. Voltage response at the terminal of the secondary cable for $\theta = 0$. (a) Transient response. (b) Amplitude frequency characteristic.
lengths of cable and different preliminary phases ($\theta$) during switching operation.

If the secondary cable is 50 m in length and the preliminary phases are $\theta = 0, \pi/2$, the voltage responses calculated by use of the proposed TDFE method in time and frequency domains are showed in Figs. 13 and 14. It can be seen from Table I and Figs. 13 and 14 that the simulated results vary with different preliminary phase angles. For example, the interference voltage can reach the maximum value at $\theta = 0$ and the minimum value at $\theta = \pi/2$. The peak-to-peak interference voltage can be as high as 25 kV if the secondary cable is 50 m in length, and this calculated value is close to that in [1]. In addition, we can also see from Figs. 13 and 14 that the main frequency spectrum is below 10 MHz.

V. CONCLUSION

An effective TDFE method is presented for the analysis of transient response of the MTLs with lumped parameter. The advantages of the TDFE method mainly include the following: 1) it can be used to solve the MTLs with lumped parameter, which is difficult to solve for the FDTD method; 2) the TDFE method can effectively suppress the instability caused by the FDTD method; and 3) the TDFE method can be used for the calculation of the whole wave processes of the voltage and current along the MTLs; however, this calculation is not effective to perform by using the software EMTP. The feasibility and effectiveness of the proposed TDFE method have been demonstrated by some experimental results. At last, the proposed TDFE method is applied to the numerical prediction of the EMI in secondary cable in substations. The proposed TDFE method will have potential applications to the analysis for transient EM fields in power systems.

REFERENCES

Lei Liu was born in Henan, China, in 1978. He received the B.S. and Ph.D. degrees from North China Electric Power University, Baoding, Hebei, China, in 1999 and 2008, respectively. He is currently an Engineer in the Technology Research Center, China Southern Power Grid, Guangzhou, China. His research interests include electromagnetic field numerical computation, electromagnetic environment on power systems, and ultrahigh-voltage power transmission technology.

Xiang Cui (M’97–SM’98) was born in Baoding, China, in 1960. He received the B.S. and M.S. degrees in theoretical electrical engineering from North China Electric Power University (NCEPU), Beijing, China, in 1982 and 1984, respectively, and the Ph.D. degree in accelerator physics from the China Institute of Atomic Energy, Beijing, in 1988. He is currently a Professor and the Head of the Electromagnetic Fields and Electromagnetic Compatibility Laboratory, NCEPU, Baoding. He is the author or coauthor of more than 100 journal articles. His current research interests include computational electromagnetics, electromagnetic environment and electromagnetic compatibility in power systems, insulation, and magnetic problems in high voltage apparatus.

Prof. Cui is a Standing Council Member of the China Electrotechnical Society, a Senior Member of IEEE and a Fellow of IET, a member of CIGRE C4.02.01 Working Group (Electromagnetic Compatibility in power systems). He is also an Associate Editor of the IEEE TRANSACTIONS ON ELECTROMAGNETIC COMPATIBILITY and a member of the Editorial Advisory Board of the International Journal for Computation and Mathematics in Electrical and Electronic Engineering (COMPEL).

Lei Qi was born in Henan, China, in 1978. He received the B.S. and M.S. degrees from North China Electric Power University, Baoding, Hebei, China, in 2000, 2003, and 2006, respectively. He is currently an Associate Professor of electrical and electronic engineering at North China Electric Power University, Beijing, China. His research interests include electromagnetic (EM) field numerical computation, EM compatibility on power systems, and ultrahigh-voltage power transmission technology.