Optimal Characteristics of an Arbitrary Receive Antenna

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***Abstract*—Several fundamental questions about the operation of receiving antennas are addressed, such as "Why does a receiving antenna scatter an incident field?" and "Under what conditions does a receive antenna capture all of the available incident power?" A new method is described by which the received power can be maximized for an arbitrary receiving antenna. The technique is first illustrated for two-dimensional infinite receiving arrays of electric and/or magnetic dipole elements, which result in simple plane waves for the scattered (re-radiated) fields. Optimal results (for maximum received power) are derived for several cases, and it is established that half the available incident power may be received by an array of electric (or magnetic) elements in free space, and that all available incident power may be received by an array that combines electric and magnetic elements, or one that incorporates a ground plane. Next, an arbitrary finite three-dimensional antenna enclosed by a mathematical spherical surface is treated using spherical vector wave functions. It is shown that half the available incident power can be received by such an antenna consisting of either TM or TE only elements, while all available incident power can be received when both TM and TE elements are used. It is also shown that the absorption efficiency for any optimal arbitrary antenna is 50%.**

***Index Terms*—Antenna optimization, antennas, receive antennas.**

I. Introduction

MOST of the fundamental limits that have been derived for antennas relate to the transmit case. These include limits on the Q and bandwidth of electrically small antennas, antenna efficiency, and the maximum gain of an arbitrary antenna [1]–[6]. While some of the characteristics of transmit antennas can be applied to receive antennas, there is merit in studying receiving antennas from first principles, especially in regard to the mechanism of the fields that are re-radiated by a receive antenna, and the fundamental limits on the maximum amount of power that can be received by a conjugately matched antenna. This paper addresses these topics by using a novel analysis to maximize the received power of an arbitrary antenna under plane wave excitation, and applying this method to specific cases that include two-dimensional (2D) infinite arrays of electric and/or magnetic elements, and to an arbitrary three-dimensional (3D) antenna of finite (but not necessarily small) size.

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The theoretical operation of general receiving antennas remains a subject that attracts much discussion, as evidenced by a number of recent articles on the subject [7]–[10]. Much of the debate involves the use (or misuse) of the Thévenin equivalent circuit, as well as differences in terminology, such as the definition of "scattered power," and its calculation.

Typically the scattering characteristics of a receiving antenna are of little interest when only received voltage and power values are desired. This is mainly due to the fact that computing received voltage and power can easily be accomplished by using the vector effective height, the effective receiving aperture, and the impedance and polarization mismatch factors. However, there seems to be no clear way of addressing these quantities for general antennas without defining the antenna structure and port configuration details.

Similar to those in numerous transmit antenna analyses [1]–[6], a general field expression can be assumed on a closed surface for the incident and scattered fields. An application of the Poynting theorem will give the power delivered to the load connected to the enclosed lossless antenna. The total field on the closed surface is expressed as a superposition of appropriate vector basis functions. In this approach the received power can be quantified in terms of the expansion coefficients without having to define any detailed information about the antenna, its terminals, or the load. Effectively, the original antenna receiving problem is re-cast into an antenna scattering problem. The receiving characteristics can then be optimized in terms of the field expansion coefficients.

II. Two Dimensional Receive Antenna Arrays

A. An Infinite Receive Array of Electric Dipoles

The simplest receive antenna case to consider is an infinite planar array of electric dipoles terminated with dissipative loads, oriented along the x-axis, and excited by a normally incident linearly polarized plane wave. We assume that the array spacing is $a \times b$ in the x and y directions, and is such that no grating lobes exist at the operating frequency. The full electromagnetic analysis of this problem requires a doubly infinite summation of propagating and evanescent Floquet modes [11], but because we are only interested in real power it is sufficient to consider only the (0,0) Floquet mode, which amounts to propagating plane waves on each side of the dipole array. The geometry of the problem is shown in Fig. 1.

Let the incident field be an x-polarized plane wave incident from the positive z direction:

$$\overline{E}_i = \hat{x}E_0 e^{jkz} \tag{1a}$$

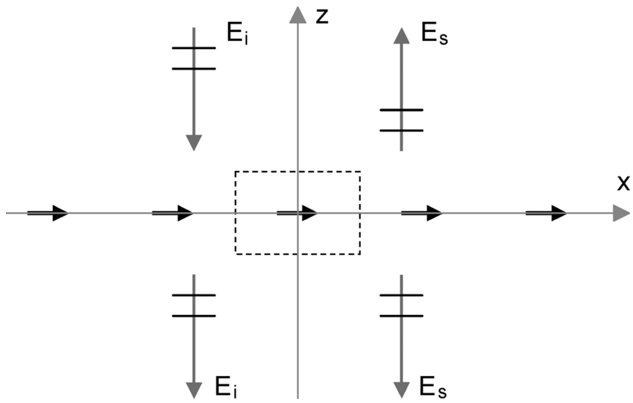

Fig. 1. Geometry of an infinite dipole array in free-space excited by a plane wave. The dipole array extends infinitely far in the x and y directions, and has dissipative loads (not shown).

$$\overline{H}_i = -\hat{y}\frac{E_0}{\eta}e^{jkz} \tag{1b}$$

where η is the intrinsic impedance and k is the wavenumber in free space. The infinite dipole array re-radiates (or *scatters*) plane waves on either side of the array; the electric field is continuous and symmetric while the magnetic field is discontinuous and anti-symmetric:

$$\overline{E}_s = \begin{cases} \hat{x}E_0 A e^{-jkz} & \text{for } z > 0 \\ \hat{x}E_0 A e^{jkz} & \text{for } z < 0 \end{cases} \tag{2a}$$

$$\overline{H}_s = \begin{cases} \hat{y}\frac{E_0}{\eta} A e^{-jkz} & \text{for } z > 0 \\ -\hat{y}\frac{E_0}{\eta} A e^{jkz} & \text{for } z < 0. \end{cases} \tag{2b}$$

Because of the periodic uniformity of the infinite array fields, the real power dissipated in one unit cell (by one dipole load) of the array can be found by integrating the real part of the Poynting vector dotted with the inward pointing unit normal vector of a surface that encloses one unit cell of the array. This amounts to computing the power flow into the top side of the enclosing surface due to the incident field, and subtracting the power flow out of the top and bottom sides of the surface. Note that the *total* field (incident plus scattered) should be used when computing the power flow out of the bottom side of the surface. Thus, the real power dissipated in the dipole load of a single unit cell can be found as

$$\begin{aligned} P_L &= \frac{1}{2}\oint\!\!\!\oint \mathrm{Re}\left\{\overline{E}_T \times \overline{H}_T^*\right\}\cdot \hat{n}ds \\ &= \frac{ab}{2\eta}|E_0|^2\left(1 - |A|^2 - |1+A|^2\right) \end{aligned} \tag{3}$$

where the integration is over the surface that encloses one unit cell.

In order the maximize this quantity it is helpful to express the (complex) scattered field coefficient in terms of its real and imaginary parts: $A = \alpha + j\beta$. Then (3) becomes

$$P_L = \frac{ab}{2\eta}|E_0|^2\left[1 - \alpha^2 - \beta^2 - (1+\alpha)^2 - \beta^2\right]. \tag{4}$$

Differentiating with respect to α and β in order to maximize the load power gives the result that,

$$A = -\frac{1}{2} \tag{5}$$

for which the maximum value of the load power is

$$P_L^{\max} = \frac{ab|E_0|^2}{4\eta}\ (\mathrm{W}). \tag{6}$$

The power density of the incident plane wave is

$$S^{\mathrm{inc}} = \frac{|E_0|^2}{2\eta} \quad (\mathrm{W/m^2}). \tag{7}$$

Defining the effective receive aperture area in the usual way gives the maximum effective aperture area of a unit cell of the receive array as

$$A_e^{\max} = \frac{P_L^{\max}}{S^{\mathrm{inc}}} = \frac{ab}{2}. \tag{8}$$

Alternatively, the ratio of load power to incident power (the receive aperture efficiency) over a single unit cell is,

$$\eta_{\mathrm{eff}} = \frac{P_L^{\max}}{P^{\mathrm{inc}}} = \frac{P_L^{\max}}{abS^{\mathrm{inc}}} = \frac{1}{2} \tag{9}$$

indicating that the dipole array can, at best, receive only half of the available incident power.

Power conservation is satisfied over each unit cell, as can be seen by computing the power densities of the plane waves propagating away from the array in the positive and negative z directions

$$S^+ = \frac{|E_0 A|^2}{2\eta} = \frac{|E_0|^2}{8\eta} \quad (\mathrm{W/m^2}) \tag{10a}$$

$$S^- = \frac{|E_0(1+A)|^2}{2\eta} = \frac{|E_0|^2}{8\eta} \quad (\mathrm{W/m^2}). \tag{10b}$$

Thus, $S^{\mathrm{inc}} = S^+ + S^- + P_L/ab$. Note that it would be incorrect to calculate a "scattered" power in the negative z direction using only the scattered fields of (2)—the total field in the region below the array should be used when computing power.

This simple problem serves to illustrate the key principle of the receiving antenna: the antenna radiates a field to partially cancel the incident field in the forward $(-z)$ direction. Because of symmetry, the electric field radiated by the dipole array has the same amplitude in the $+z$ and $-z$ directions, so the best result is to cancel only half the incident field in the $-z$ direction, since an equivalent field is generated in the $+z$ direction. Thus, half of the available incident power is re-radiated by the antenna, and half is absorbed by the dipole loads. If the scattered field amplitude were larger, e.g., equal to the (negative) of the incident field, then the total field propagating away from the array in the $-z$ direction would be zero, but the scattered field component propagating in the $+z$ direction would lead to a power density that is four times that of (10b), and the power delivered to the dipole loads would be zero. Similarly, if the scattered field amplitude were smaller than the optimum value given in (5), less of the incident field would be canceled in the $-z$ direction, and the net power delivered to the loads would be less than that given in (6).

B. An Infinite Receive Array of Electric and Magnetic Dipoles

Next consider an infinite planar receiving array consisting of electric and magnetic dipoles (e.g., small dipoles and small loops), polarized in the x-direction, again excited with the normally incident linearly polarized plane wave of (1). The geometry is the same as that of Fig. 1, but with the addition of magnetic dipoles oriented in the y-direction. Assuming the absence of grating lobes, the propagating fields on either side of the array can be written as

$$\overline{E}_s = \begin{cases} \hat{x}E_0(A+B)e^{-jkz} & \text{for } z>0 \\ \hat{x}E_0(A-B)e^{jkz} & \text{for } z<0 \end{cases} \tag{11a}$$

$$\overline{H}_s = \begin{cases} \hat{y}\frac{E_0}{\eta}(A+B)e^{-jkz} & \text{for } z>0 \\ -\hat{y}\frac{E_0}{\eta}(A-B)e^{jkz} & \text{for } z<0 \end{cases} \tag{11b}$$

where A is the amplitude of the field radiated by the electric dipoles, and B is the amplitude of the field radiated by the magnetic dipoles. Note that the electric dipoles radiate a symmetric electric field on either side of the array, while the magnetic dipoles radiate an anti-symmetric electric field.

Following the same procedure as in Section II-A, we compute the real power lost in the antenna loads of one unit cell as

$$P_L = \frac{ab}{2\eta}|E_0|^2\left(1-|A+B|^2-|1+A-B|^2\right). \tag{12}$$

Maximizing by differentiating with respect to the real and imaginary parts of A and B gives the optimal values for the wave amplitudes as

$$A = -\frac{1}{2} \tag{13a}$$

$$B = \frac{1}{2}. \tag{13b}$$

The maximum value of load power is then

$$P_L^{\max} = \frac{ab|E_0|^2}{2\eta}. \tag{14}$$

The power density of the incident plane wave is given in (7), so the maximum effective aperture area of a unit cell of the combined electric/magnetic dipole array is

$$A_e^{\max} = \frac{P_L^{\max}}{S^{\text{inc}}} = ab. \tag{15}$$

This result is seen to be twice that obtained for the electric dipole-only case, indicating that the combination of electric and magnetic dipoles can function to receive twice as much of the available incident power. This occurs because of the anti-symmetry introduced by the addition of the magnetic dipoles, which makes it possible for the array to re-radiate a scattered field that is greater in the $-z$ direction than in the $+z$ direction, thus allowing complete cancellation of the incident field in the forward direction without generating any propagating wave in the backward direction. This can be seen by using (11) and (13) to compute the power densities of the total plane waves propagating away from the array in the positive and negative z directions

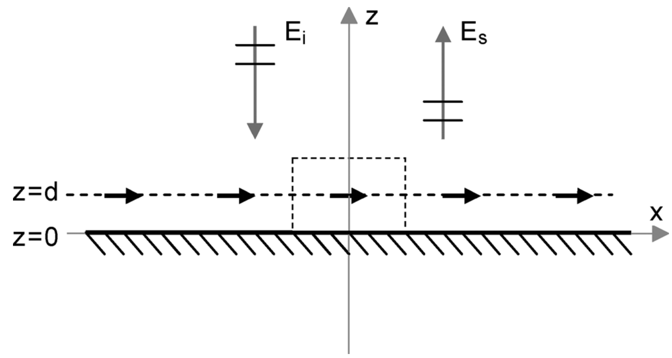

Fig. 2. Geometry of an infinite dipole array over a perfectly conducting ground plane. The dipole array extends infinitely far in the x and y directions, and has dissipative loads (not shown).

$$S^+ = \frac{|E_0(A+B)|^2}{2\eta} = 0 \tag{16a}$$

$$S^- = \frac{|E_0(1+A-B)|^2}{2\eta} = 0. \tag{16b}$$

Power conservation is obviously satisfied. Again, it would be incorrect to refer to the *power* scattered by the array, as computed from the fields of (11), without superposition of the incident field.

Setting $B = 0$ in this solution recovers the results of Section II-A, while setting $A = 0$ provides the solution for an array consisting only of magnetic dipoles (small loops), and yields the same results for maximum effective area as given in (8) for the electric dipole case.

C. An Infinite Receive Array of Electric Dipoles Over a Ground Plane

Next consider an infinite planar receiving array of electric dipoles, in the $z = d$ plane, positioned over a perfect ground plane in the $z = 0$ plane, as shown in Fig. 2. Again let the dipoles be x-polarized, and excited by a plane wave from the $+z$ direction. In this case, however, we choose to define the incident field as the field that exists in the absence of the dipoles, but in the presence of the ground plane. Thus, we have

$$\overline{E}_i = \hat{x}E_0(e^{jkz}-e^{-jkz}) \tag{17a}$$

$$\overline{H}_i = -\hat{y}\frac{E_0}{\eta}(e^{jkz}+e^{-jkz}). \tag{17b}$$

The field scattered by the dipole array can be written as,

$$\overline{E}_s = \hat{x}E_0A(1-e^{-j2kd})e^{-jkz} \text{ for } z>d \tag{18a}$$

$$\overline{H}_s = \hat{y}\frac{E_0}{\eta}A(1-e^{-j2kd})e^{-jkz} \text{ for } z>d \tag{18b}$$

where A is a complex amplitude. The field in the region between the dipoles and the ground plane is not needed for this analysis.

The power dissipated in the load of the dipole of one unit cell can be found by subtracting the power density of the total plane wave field radiated outward from the unit cell from the power density of the plane wave impinging upon the unit cell. The former involves the scattered field of (18) and the field of the second term in (17), while the latter involves only the field of the first term of (17)

$$\begin{aligned} P_L &= ab(S^- - S^+) \\ &= \frac{ab|E_0|^2}{2\eta}\left(1 - |-e^{-jkd} + Ae^{-jkd} - Ae^{-j3kd}|^2\right) \\ &= \frac{ab|E_0|^2}{2\eta}\left(1 - |1 - 2A|^2\right). \end{aligned} \tag{19}$$

The last two lines of (19) incorporate the fact that we are setting $d = \lambda/4$ to simplify results.

Maximizing the load power gives the optimal amplitude coefficient for the scattered power as,

$$A = \frac{1}{2} \tag{20}$$

and the maximum power delivered to the load as

$$P_L^{\max} = \frac{ab|E_0|^2}{2\eta} \tag{21}$$

indicating that all of the available power from the incident plane wave may be received by the dipole array over a ground plane, and the maximum effective aperture area is the same as given in (15). This result is in agreement with the conclusions of several other authors [10], [12], [13]. Again, it is easy to verify that power conservation is satisfied, and that the net field propagating away from the array and ground plane, in the $+z$ direction, is zero. Carrying through the analysis for general ground plane spacing, d, leads to the same results given in (20) and (21), except for limiting cases when d is a multiple of a half wavelength.

D. *Other Cases*

There are a number of related infinite array cases that can be studied in the same manner as those above. Oblique incidence can be easily treated, for either vertical or horizontal polarization, with the result that the maximum effective aperture area of the array will vary as $\cos\theta$, where θ is the angle measured from the z-axis. Circularly polarized arrays using crossed electric dipoles in free space will perform similarly to the linearly polarized dipole case, with a maximum receive aperture efficiency of 50%. Placing the dipoles over a ground plane will increase the aperture efficiency to 100%. It is also possible to achieve circular polarization using a combination of electric and magnetic elements (TM and TE modes), in which case 100% maximum receive aperture efficiency will be achieved. Using two parallel arrays of dipoles introduces an extra degree of freedom, again allowing the aperture efficiency to reach 100%.

III. Arbitrary Three Dimensional Receive Antenna

A. *Received Power in Terms of Spherical Vector Wave Functions*

To analyze the receiving characteristics of an arbitrary single antenna of finite dimensions in free space, it is assumed that a spherical surface S of radius a encloses the receiving antenna.

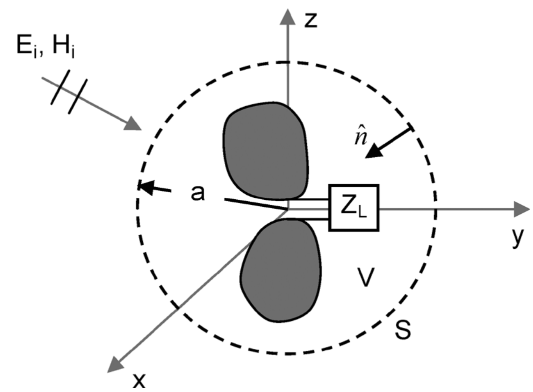

Fig. 3. An arbitrary 3D antenna of finite dimensions subject to the incident field $\overline{E}_i, \overline{H}_i$. The antenna is enclosed within a sphere of radius a.

This is illustrated in Fig. 3, where the center of S is located at the coordinate origin, and the antenna is subject to the arbitrarry incident fields $\overline{E}_i, \overline{H}_i$. The choices of the geometry of the closed surface S and the associated coordinate system are arbitrary. A spherical system is chosen here for convenience of the analysis that follows.

The total fields are expressed as the vector sum of the incident and the scattered fields, i.e., $\overline{E}_T = \overline{E}_i + \overline{E}_s$ and $\overline{H}_T = \overline{H}_i + \overline{H}_s$. The incident fields are defined as the total fields in the absence of the receiving antenna. When the antenna receives power, the incident fields induce electric and/or equivalent magnetic currents over the antenna structure. The induced current flowing through the antenna terminals delivers power to the attached load. In addition to delivering power to the load, the induced currents re-radiate, producing the scattered fields $\overline{E}_s$ and $\overline{H}_s$. Knowledge of the total tangential fields over S uniquely determines the power lost inside S (within V) via (3). If the antenna is lossless, all the power lost within V is delivered to the load. Therefore, the power P_L delivered to the load can be completely specified in terms of the incident and the scattered field quantities over S.

The electric and magnetic fields can be expanded in terms of spherical vector wave harmonics. These vector harmonic wave functions $\overline{M}_q^{(i)}$ and $\overline{N}_q^{(i)}$ are defined in [14]. The index $q = {}^{e}_{o}mn$ is the compounded index for notational simplicity, and the superscript (i) indicates the type of radial dependence for the vector wave functions. The corresponding radial function is given by $z_n^{(i)} = j_n, y_n, h_n^{(1)}, h_n^{(2)}$ for $i = 1, \ldots, 4$, respectively, chosen depending on the region of interest. Both the incident fields and the scattered fields may be expressed in terms of $\overline{M}_q$ and $\overline{N}_q$. Arbitrary incident fields $\overline{E}_i, \overline{H}_i$ may be expanded as

$$\overline{E}_i = E_0 \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(A_q^i \overline{M}_q^{(1)} + B_q^i \overline{N}_q^{(1)}\right) \tag{22a}$$

$$\overline{H}_i = \frac{jE_0}{\eta} \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(B_q^i \overline{M}_q^{(1)} + A_q^i \overline{N}_q^{(1)}\right) \tag{22b}$$

where A_q^i, B_q^i are expansion coefficients for the associated TE and TM modes, respectively. The requirement of finite-valued fields at the origin requires the use of the spherical Bessel function $(z_n = z_n^{(1)} = j_n)$ for the radial coordinate. Equation (22) is valid at any distance r from the origin. It is noted that these expressions are general for describing incident fields having arbitrary polarization and having spectral contents ranging from propagating to evanescent components. In the range $r \geq a$, the expansions for the scattered fields can be expressed as

$$\overline{E}_s = E_0 \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(A_q^s \overline{M}_q^{(4)} + B_q^s \overline{N}_q^{(4)} \right) \tag{23a}$$

$$\overline{H}_s = \frac{jE_0}{\eta} \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(B_q^s \overline{M}_q^{(4)} + A_q^s \overline{N}_q^{(4)} \right) \tag{23b}$$

in terms of complex expansion coefficients A_q^s, B_q^s. In principle, these coefficients can be found as the inner products between the associated vector mode functions and the antenna induced currents. Since the scattered fields propagate radially outward, the spherical Hankel function of the 2nd kind is chosen for the radial dependence $(z_n = z_n^{(4)} = h_n^{(2)})$.

Employing the surface integral of the Poynting vector over S as in (3), an expression for P_L may be found in terms of the expansion coefficients and a. Due to the orthogonality between different modes and orders, the total delivered power is the sum of absorbed powers from each mode and order

$$P_L = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(P_q^{\mathrm{TE}} + P_q^{\mathrm{TM}} \right) \tag{24}$$

where

$$\begin{aligned} P_q^{\mathrm{TE}} =& \mathrm{Re} \frac{j|E_0|^2}{2\eta} \\ &\times \oiint_S \Big(A_q^i A_q^{i*} \overline{M}_q^{(1)} \times \overline{N}_q^{(1)*} + A_q^i A_q^{s*} \overline{M}_q^{(1)} \times \overline{N}_q^{(4)*} \\ &\qquad + A_q^s A_q^{i*} \overline{M}_q^{(4)} \times \overline{N}_q^{(1)*} + A_q^s A_q^{s*} \overline{M}_q^{(4)} \times \overline{N}_q^{(4)*} \Big) \\ &\qquad \cdot \hat{r} ds \end{aligned} \tag{25a}$$

$$\begin{aligned} P_q^{\mathrm{TM}} =& \mathrm{Re} \frac{j|E_0|^2}{2\eta} \\ &\times \oiint_S \Big(B_q^i B_q^{i*} \overline{N}_q^{(1)} \times \overline{M}_q^{(1)*} + B_q^i B_q^{s*} \overline{N}_q^{(1)} \times \overline{M}_q^{(4)*} \\ &\qquad + B_q^s B_q^{i*} \overline{N}_q^{(4)} \times \overline{M}_q^{(1)*} + B_q^s B_q^{s*} \overline{N}_q^{(4)} \times \overline{M}_q^{(4)*} \Big) \\ &\qquad \cdot \hat{r} ds. \end{aligned} \tag{25b}$$

Specifically, the orthogonality relationships for the radial component of the cross product between $\overline{M}_q$ and $\overline{N}_q$ are

$$\begin{aligned} &\oiint_S \overline{M}_q^{(i)} \times \overline{M}_{q'}^{(j)*} \cdot \hat{r} ds \\ &\quad = \oiint_S \overline{N}_q^{(i)} \times \overline{N}_{q'}^{(j)*} \cdot \hat{r} ds = 0, \text{ all } q, q' \end{aligned} \tag{26a}$$

$$\begin{aligned} &\oiint_S \overline{M}_q^{(i)} \times \overline{N}_{q'}^{(j)*} \cdot \hat{r} ds \\ &\quad = \oiint_S \overline{N}_q^{(i)} \times \overline{M}_{q'}^{(j)*} \cdot \hat{r} ds = 0, \quad q \neq q' \end{aligned} \tag{26b}$$

$$\begin{aligned} &\oiint_S \overline{M}_q^{(i)} \times \overline{N}_q^{(j)*} \cdot \hat{r} ds \\ &\quad = \frac{a}{k} \lambda_{mn} z_n^{(i)}(ka) \\ &\quad \times \left[z_n^{(j)}(ka) + ka z_n^{(j)\prime}(ka) \right]^*, \quad q = q' \end{aligned} \tag{26c}$$

where $\lambda_{mn} = (2\pi\varepsilon_m/(2n+1))n(n+1)((n+m)!/(n-m)!)$, and $\varepsilon_m = 2$ when $m = 0$ and $\varepsilon_m = 1$ otherwise. The prime on the Bessel function denotes differentiation with respect to the whole argument. Equations (26a)–(26c) are valid for any combination of $(i, j) = (1, 1)$, (1, 4), (4, 1), or (4, 4).

The complete expressions for P_q^{TE} and P_q^{TM} are found to be

$$\begin{aligned} P_q^{\mathrm{TE}} &= \frac{|E_0|^2}{2\eta} \frac{a\lambda_{mn}}{k} \\ &\quad \times \mathrm{Re} \Big\{ A_q^{i*} A_q^s y_n (j_n + ka j_n') \\ &\qquad - A_q^i A_q^{s*} j_n (y_n + ka y_n') - \frac{1}{ka} \left| A_q^s \right|^2 \Big\} \\ &= -\frac{|E_0|^2}{2k^2\eta} \lambda_{mn} \left(\mathrm{Re} \left\{ A_q^{i*} A_q^s \right\} + \left| A_q^s \right|^2 \right) \end{aligned} \tag{27a}$$

$$P_q^{\mathrm{TM}} = -\frac{|E_0|^2}{2k^2\eta} \lambda_{mn} \left(\mathrm{Re} \left\{ B_q^{i*} B_q^s \right\} + \left| B_q^s \right|^2 \right). \tag{27b}$$

The permissible values of A_q^s and B_q^s are dictated by the fact that there is no net power escaping S for each mode and order. One finds that $P_q^{\mathrm{TE}} \geq 0$ and $P_q^{\mathrm{TM}} \geq 0$ translate into the following set of conditions:

$$\left| A_q^s + \frac{1}{2} A_q^i \right| \leq \frac{1}{2} \left| A_q^i \right| \tag{28a}$$

$$\left| B_q^s + \frac{1}{2} B_q^i \right| \leq \frac{1}{2} \left| B_q^i \right|. \tag{28b}$$

The total received power in (24) can now be written as

$$\begin{aligned} P_L = -\frac{|E_0|^2}{2k^2\eta} \sum_{n=1}^{\infty} \sum_{m=0}^{n} \lambda_{mn} \Big(&\mathrm{Re} \left\{ A_q^{i*} A_q^s \right\} + \mathrm{Re} \left\{ B_q^{i*} B_q^s \right\} \\ &+ \left| A_q^s \right|^2 + \left| B_q^s \right|^2 \Big). \end{aligned} \tag{29}$$

The maximum value is equal to

$$P_L^{\max} = \frac{|E_0|^2}{8k^2\eta} \sum_{n=1}^{\infty} \sum_{m=0}^{n} \lambda_{mn} \left(\left| A_q^i \right|^2 + \left| B_q^i \right|^2 \right) \tag{30}$$

which is achieved for the following values of scattered field coefficients:

$$A_q^s = -\frac{1}{2} A_q^i \tag{31a}$$

$$B_q^s = -\frac{1}{2} B_q^i. \tag{31b}$$

It is interesting to inspect the radial dependence of the total fields under the optimum receiving condition (31). Noting that the spherical Bessel function for the incident fields represents a superposition of radially converging and diverging waves, one finds that the radial function for each mode of the total fields $\overline{E}_T$, $\overline{H}_T$ reduces to

$$j_n - \frac{1}{2}h_n^{(2)} = \frac{1}{2}\left[h_n^{(1)} + h_n^{(2)}\right] - \frac{1}{2}h_n^{(2)} = \frac{1}{2}h_n^{(1)}. \tag{32}$$

This indicates that the optimum receiving antenna exactly cancels all outward-traveling wave components in the incident fields, so that the total fields are composed purely of radially converging waves in $r > a$. This is analogous to the 100% power absorption by a 2D array of electric/magnetic dipoles presented in Section II-B, where the incident field is composed of the incoming $(z > 0)$ and the outgoing $(z < 0)$ waves from the array's point of view $(z = 0)$. The combined scattered fields from the electric and magnetic dipoles add to the incident fields to produce null fields in the forward direction, while no scattering occurs at all in the backward direction. When the incident fields contain both TE and TM modes, satisfaction of either (31a) or (31b) by a 3D receiving antenna will result in a configuration analogous to the 2D scenario discussed in Section II-A. The outward-travelling components contained in the incident fields are partially canceled by the scattered fields.

Qualitatively, the receive process of an antenna can be described as follows. Under an illumination by an incident field, surface/volume currents of electric and/or magnetic type are induced over the antenna structure. These currents satisfy the appropriate boundary conditions on the antenna structure, e.g., the tangential electric field over the conducting surfaces being equal to zero. Across the receive terminals, the voltage and current satisfy the appropriate load condition dictated by the load impedance. This load condition affects the induced current distribution. The coefficients A_q^s and B_q^s describe how much the corresponding TE and TM spherical wave components are excited by the induced currents. Thus, they are functions of the attached load via the induced currents. Equation (31) is the condition on these coefficients such that the maximum power is delivered to the load through the antenna terminal. Maximum received power generally implies a conjugately matched load.

B. *Plane Wave Reception*

The maximum delivered power (30) was derived for general incident fields, and is not limited to plane waves. For example, illumination by a source in the Fresnel zone of the receive antenna can be easily treated. Still, plane wave illuminations constitute the most important class of receive antenna configurations.

Assume that the receiving antenna is illuminated by a plane wave that is linearly polarized in the $\hat{x}$ direction and propagates in the $-\hat{z}$ direction as described by (1). Its spherical wave decomposition can be written as [14]

$$A_{omn}^i = \delta_{m1} j^n \frac{2n+1}{n(n+1)}, \quad A_{emn}^i = 0 \tag{33a}$$

$$B_{omn}^i = 0,\ B_{emn}^i = -\delta_{m1} j^{n+1} \frac{2n+1}{n(n+1)} \tag{33b}$$

where δ is the Kronecker delta. Equation (33) indicates that only the $m = 1$ harmonics are non-zero in the ϕ coordinate. More importantly, it implies that a plane wave, which is a TEM wave, contains exactly equal amounts of TE and TM spherical mode components for each order. The maximum receive power for plane wave illumination is found from (30) to be

$$P_L^{\max} = \frac{\pi |E_0|^2}{2k^2\eta} \sum_{n=1}^{\infty} (2n+1) \tag{34}$$

which clearly diverges to infinity. The implication is that the maximum power received by an antenna enclosed within a finite spherical volume of radius a does not have an upper bound, as long as the induced currents excite the scattered spherical wave components of arbitrarily high orders.

The resulting total fields in the forward and the backward directions can be computed from the field expressions in the two directions $\theta = \pi$, 0. After some algebraic manipulation using (23), (32) and (33), employing the values of the Legendre functions at these angles [6], and the large argument forms of the various Bessel functions, one obtains (for $r \gg a$)

$$\overline{E}_T(r, \theta = \pi) = \overline{0} \tag{35a}$$

$$\overline{E}_T(r, \theta = 0) = \hat{x} E_0 e^{jkz}. \tag{35b}$$

Thus we see that the optimum receive antenna produces a particular scattering pattern such that the total field reduces to zero in the forward direction and stays unchanged from the incident field in the backward direction. An exact satisfaction of (35) is achieved only when an infinite number of spherical modes are excited for the scattered fields by the antenna, and will be approximately satisfied if a finite number of low-order spherical modes are involved. As more higher-order modes are properly excited, $\overline{E}_s$, $\overline{H}_s$ will become closer to $-\overline{E}_i$, $-\overline{H}_i$ for $z < 0$, canceling the incident field, and producing a null backscattered field for $z > 0$.

Using the usual argument that spherical modes of order $N > ka$ decay quickly [1]–[3], we let the infinite series in (34) be limited to order N. Evaluating the finite series, one obtains

$$P_L^{\max} = \frac{\pi |E_0|^2}{2k^2\eta} (N^2 + 2N) \tag{36}$$

which has the associated maximum effective aperture $A_e^{\max}$ equal to

$$A_e^{\max} = \frac{\lambda^2}{4\pi}(N^2 + 2N). \tag{37}$$

Via reciprocity, (37) indicates that the maximum gain G_0 of the antenna in the transmitting mode is equal to $G_0 = N^2 + 2N$, which agrees with the result obtained for the transmit mode analysis [3].

Now consider an electrically large receiving antenna that receives both TE and TM spherical modes. In this case, the maximum spherical mode order will be $N \approx ka \gg 1$. From (37), the maximum effective aperture is given by

$$A_e^{\max} \approx \frac{\lambda^2}{4\pi}(ka)^2 = \pi a^2 \tag{38}$$

which is equal to the physical cross-sectional area of the sphere circumscribing the receiving antenna. Therefore, such an antenna that receives both TE and TM modes has a maximum receiving aperture efficiency of 100%. However, if the antenna receives either TE or TM modes only, the aperture efficiency will be reduced to 50%. Comparing (30) and (33), it can be seen that sub-optimal receiving performance of TE-mode or TM-mode only antennas is due to the fact that a plane wave contains an equal amount of TE and TM spherical mode power. Conversely, if the incident field contains either TE or TM modes only, an antenna capable of receiving either TE or TM modes only will be the optimum receiver.

C. Limits for Small Antennas

To investigate the receiving property in the small antenna limit, consider an electrically small antenna ($ka \ll 1$) such that only the fundamental TE and TM modes are induced on the antenna, i.e., $N = 1$ in (37). In this case, the maximum received power is

$$P_L^{\max} = \frac{\pi |E_0|^2}{2k^2\eta} \times 3 = \frac{|E_0|^2}{2\eta}\frac{3\lambda^2}{4\pi} \tag{39}$$

associated with the gain of 3 in the transmit mode, and a maximum effective receive aperture area of

$$A_e^{\max} = \frac{3\lambda^2}{4\pi}. \tag{40}$$

For linear polarizations, the optimum receiver is provided by the Huygens' source [5]. A small antenna realization in terms of a dipole-loop combination has been reported by Green [15]. For circular polarizations, a combination of four elementary electric and magnetic linear antennas is capable of receiving this maximum power [16].

An elementary electric or magnetic dipole antenna will receive a maximum of 50% of the power given in (39), and exhibit half the effective area given in (40). This still assumes a perfect polarization match, which illustrates the point that the scattered wave coefficients should satisfy (31) for the proper order as well as the mode to receive power optimally. For example, a $\hat{y}$-directed infinitesimal dipole will not be able to receive power from the $\hat{x}$-polarized incident plane wave given by (33). This TM-mode antenna is capable of creating a non-zero value for B_{o11}^s only, but the incident field does not contain the TM mode of this particular order.

In summary, an optimal finite receiving antenna tries to cancel all the radially inward-traveling spherical wave components contained in the incident field that it is capable of doing. If the induced electric and magnetic currents over the antenna can excite a finite set of spherical modes and orders for the scattered field, the optimal receive operation will adjust the associated coefficients for the scattered field according to (31). The maximum received power will be given by (30), summed over the supported modes and orders.

D. Absorption Efficiency

For electromagnetic scatterers and antennas the scattered power is typically defined as[1]

$$P_s = \frac{1}{2}\oiint_S \mathrm{Re}\left\{\overline{E}_s \times \overline{H}_s^*\right\} \cdot \hat{r}\, ds. \tag{41}$$

In terms of the spherical vector expansion coefficients defined above, P_s is expressed as

$$P_s = \frac{|E_0|^2}{2k^2\eta}\sum_{n=1}^{\infty}\sum_{m=0}^{n}\lambda_{mn}\left(\left|A_q^s\right|^2 + \left|B_q^s\right|^2\right). \tag{42}$$

For a receive antenna the absorption efficiency $\eta_{\rm abs}$ is defined as the ratio of the power P_L delivered to the load to the sum of delivered and scattered powers [18] as in (43) at the bottom of the page, where un-optimized expressions for P_L and P_s were used. As has been found in [18], one can observe that $\eta_{\rm abs}$ may approach unity as closely as desired. However, (43) shows that this limit is achieved only by letting $|A_q^s| \ll |A_q^i|$, $|B_q^s| \ll |B_q^i|$. In other words, as the strength of the induced current over the antenna structure is reduced, the antenna becomes a poorer scatterer faster than it becomes a poorer receiver.

For an arbitrary optimal receive antenna characterized by (31), it follows from (43) that $\eta_{\rm abs} = 0.5$. If P_s in (41) is interpreted as a real power, one could say that any antenna optimized for maximum reception scatters just as much power as it absorbs. This statement applies to antennas under arbitrary incident fields, not just plane wave illuminations.

IV. Conclusion

A new general method for analyzing the receiving properties of arbitrary antennas has been presented. The need for defining specific details of the receiving antenna structure and the connected load was obviated by casting the scattered fields in terms of outward-traveling waves with arbitrary expansion coefficients. The power delivered to the load was expressed as an integral of the Poynting vector over a closed surface surrounding the antenna. The proposed technique provides a general method for analyzing an antenna's receiving properties,

[1]It should be noted that scattered power in this context does not exist as a separate measurable, independent quantity of real power that enters into any statement of power conservation [8], [17].

$$\begin{aligned}\eta_{\rm abs} &= \frac{P_L}{P_L+P_s}\\ &= \frac{\sum_{n=1}^{\infty}\sum_{m=0}^{n}\lambda_{mn}\left(\mathrm{Re}\left\{A_q^{i*}A_q^s\right\} + \mathrm{Re}\left\{B_q^{i*}B_q^s\right\} + \left|A_q^s\right|^2 + \left|B_q^s\right|^2\right)}{\sum_{n=1}^{\infty}\sum_{m=0}^{n}\lambda_{mn}\left(\mathrm{Re}\left\{A_q^{i*}A_q^s\right\} + \mathrm{Re}\left\{B_q^{i*}B_q^s\right\}\right)}\end{aligned} \tag{43}$$

subject to arbitrary incident fields, based on first principles, without relying on its transmit characteristics via reciprocity.

Using the new technique, received powers for several planar infinite receiving array configurations were investigated. It was found that symmetry or asymmetry of the scattered fields in the two directions normal to the array surface plays the critical role in determining the maximum received power under plane wave illumination. A conductor-backed electric dipole array and an array of electric/magnetic dipoles were found to be capable of receiving 100% of the available incident power. Finally, the receive property of a single arbitrary 3D antenna of finite size was analyzed. The optimum values of the spherical vector wave expansion coefficients for the scattered waves were found in terms of those for the incident waves for delivering maximum power to the load. For incident plane waves, antennas capable of receiving both TE and TM spherical modes provide the maximum received power. It was found that TE- or TM-mode only antennas receive a maximum of only 50% of the power received by the optimum antenna. It has also been established that the absorption efficiency of an arbitrary optimum receive antenna is 50%.

As far as realization of these optimal antennas, we have noted that the known results for small dipoles and loops are in agreement with those presented here for the special case of small arbitrary optimal antennas, as is the dipole-loop antenna of Green [15]. Examples for larger optimal antennas can be found in the literature [10], [17], where it is shown that finite dipole arrays as small as 3×3 or 4×4, either in free-space or over a ground plane, can provide aperture efficiencies approaching those of the corresponding infinite array cases.

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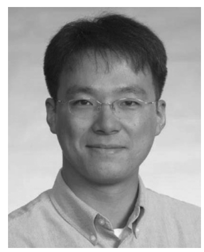

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