Rapid Method for Finding Faulty Elements in Antenna Arrays Using Far Field Pattern Samples

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Abstract—A simple and fast technique that allows a diagnosis of faulty elements in antenna arrays, that only needs to consider a small number of samples of its degraded far-field pattern is described. The method tabulates patterns radiated by the array with 1 faulty element only. Then, the pattern corresponding to the configuration of failed/undefailed elements under test is calculated using the error-free pattern and the patterns with 1 faulty element. The configuration with the lowest difference between the calculated and the degraded patterns is selected. Comparison of the performance of this method using an exhaustive search and a genetic algorithm for an equispaced linear array of 100 λ/2-dipoles is shown. Mutual coupling as well as noise/measurement errors in the pattern samples were considered in the numerical analysis.

Index Terms—Fault diagnosis, genetic algorithms, phase-array antennas.

I. INTRODUCTION

In present years, array antennas are working in several applications such as sonar, radar and communications. The antenna, in such cases, has several hundreds of radiating elements or subarrays, and the possibility of failure of some of them increases due to the high number of radiators. These element failures cause sharp variations in the field intensity across the array aperture, thus increasing both the side lobe and ripple level of the power pattern.

Array antennas have the advantage that the radiation pattern can be partially restored by changing their feeding distribution [1]. In many cases, the excitations of the non-defective elements can be readjusted to produce a pattern with a minimal loss of quality with respect to the original one. In the literature, we found several approaches that perform this compensation by numerically finding a new set of excitations of the unfailed elements that optimizes some objective function [2]–[7]. Obviously, these techniques require to know the number and the location of the failed elements in the array.

The number and location of failed elements can be inferred utilizing external data: the fault-free radiation pattern and the measurement of a given number of spatial directions of the damaged radiation pattern. Several approaches can be found in the literature. In [8] a neural network (MLP) is used to locate a maximum of three faulty elements in a 16-element array, so the results may be restricted to small arrays. In [9], a Moore-Penrose pseudoinverse of a matrix is applied to retrieve the excitation distribution of a planar array of parallel dipoles with faulty elements, by measuring the complex radiated field in its near zone. This method achieves good results at the expense of requiring lots of measurements of both the amplitude and phase of the degraded pattern. Other approaches find the solution among the search space by means of a genetic algorithm, which, based on a discrete representation of the unknowns in the case of “on-off” fault (i.e., the array elements fail completely), matches the problem at hand: Bucci et al. [10] studied the ambiguity of the solution in the continuous and the discrete “on-off” cases using amplitude-only pattern samples, and then proposed a modified genetic algorithm to solve the problem in the discrete case. In [11], a study of the influence of the number, location, and noise level of the amplitude-only samples in the performance of the genetic algorithm is presented. Although these algorithms have a high success rate in finding the faulty elements, they require computation of the array factor for each configuration under test, what can be very slow as the number of the evaluations increases.

In this paper, a simple and fast technique of diagnosis of antenna array faults using a small number of samples of its degraded far-field pattern is described. The proposed technique may be applicable together with the previous methods of array diagnosis that are based on the evaluation of the array factor (e.g., the genetic algorithms mentioned above), in order to improve its numerical efficiency.

II. THEORY

Let us consider a planar array of $N$ identical and equally oriented elements. Assuming that the elements are located on the $y = 0$ plane, the far field pattern can be calculated using the following expression [12]:

$$F(\theta, \phi) = \sum_{n=1}^{N} f_e(\theta, \phi) I_n \exp \left\{ j k (x_n \sin \theta \cos \phi + z_n \cos \theta) \right\}$$

$$= \sum_{n=1}^{N} H_n(\theta, \phi)$$

where $f_e(\theta, \phi)$ is the element pattern, $I_n$ is the relative excitation of the $n$th-element located at the position given by $(x_n, z_n)$, $k$ is the wavenumber $2\pi/\lambda$, and $H_n(\theta, \phi)$ is the directionally
weighted phasor corresponding to the \( n \)th-element at a given angular position \((\theta, \phi)\).

Let us consider that mutual coupling is not taken into account (in this model, current sources for the array elements are used). We will assume the so-called “on-off” fault, i.e., the faulty elements have zero relative excitations. Therefore, if a set \( C \) contains the indexes of the faulty elements, the degraded far field pattern radiated by this array, \( F_C \), can be obtained just by removing the phasors corresponding to failures from the summation (1)

\[
F_C(\theta, \phi) = F(\theta, \phi) - \sum_{n \in C} H_n(\theta, \phi)
\]

\[
= \sum_{n=1, n \notin C}^N H_n(\theta, \phi), \quad (2)
\]

Now, we assume that the \( N \) patterns \( \{F_n(\theta, \phi)\}_{n=1}^N \) radiated by the array when only the \( n \)th-element is failing (single-fault patterns) are known. In this case, the pattern radiated by the defective array, \( F_C \), can be calculated (only without considering mutual coupling), using the error-free pattern and the single-fault patterns \( \{F_n(\theta, \phi)\}_{n \in C} \) as in (3)

\[
F_C(\theta, \phi) = F(\theta, \phi) - \sum_{n \in C} [F(\theta, \phi) - F_n(\theta, \phi)]
\]

where \( H_n(\theta, \phi) = F(\theta, \phi) - F_n(\theta, \phi) \).

If mutual coupling is taken into account, the presence of faulty elements modifies the relative excitations of every array element and therefore the degraded pattern cannot be calculated by using (3). In this case, the exact degraded pattern must be obtained as follows: the impedance matrix is calculated [13] and then inverted to obtain the currents that are finally used to compute the pattern (1). This model, which uses voltage sources for the array elements (that are zero for the faulty/passive elements in the “on-off” fault), has been validated by means of a FEKO simulation [14] for a small array of \( \lambda/2 \)-dipoles (where the mutual coupling is stronger than for a large linear array such as the used in this paper, because in large arrays all but the elements near to the ends of the periphery “see” approximately the same environment [12]). Note that, due to the mutual coupling, the array relative excitations \( I_n \) do not keep constant values, because they depend on the configuration of the defective elements, what makes impossible the calculation, in a precise way, of the degraded field pattern by removing the phasors corresponding to failures, as in (3). Although (3) does not give the exact pattern, we have found that if the single-fault patterns \( \{F_n(\theta, \phi)\}_{n=1}^N \) are calculated by taking the mutual coupling into account, expression (3) gives a good approximation of the exact pattern radiated by the array with failures, and it stays being very useful for finding faulty elements, as Section III reports.

The procedure of locating defective elements in an antenna array begins with the measurement of the degraded pattern \( F_D(\theta, \phi) \) (emitted by the antenna presenting one or more failing elements) in \( M \) directions \( \{\theta_m, \phi_m\}_{m=1}^M \). Then, the method compares the measured radiation pattern (via the samples) with the pattern corresponding to the array with a given configuration of failed/unfailed elements. This is performed by finding that set of faulty elements \( (C) \), that minimizes the squared distance \( d_C \) between the pattern associated to this configuration \( F_C(\theta, \phi) \) and the measured pattern \( F_D(\theta, \phi) \)

\[
d_C = \sum_{m=1}^M [F_D(\theta_m, \phi_m) - F_C(\theta_m, \phi_m)]^2. \quad (4)
\]

As stated above, some of the methods described in the literature [10], [11] use a genetic algorithm to find the configuration \( C \). In this case, a chromosome contains a binary encoding of the array elements that describe the status (failed/unfailed) of each array element. However, these methods are very slow, because they use the array factor (1) every time it is necessary to compute the pattern associate to each configuration/chromosome under test.

The method proposed in this paper begins using expression (1) to tabulate both the error-free pattern as well as the \( N \) single-fault patterns \( \{F_n(\theta_m, \phi_m)\}_{n=1}^N \) evaluated at all the \( M \) directions of the provided samples. Then, computation of \( d_C \) in (4) for a given configuration \( C \) of faulty elements is very fast, because \( F_C \) is calculated just by using the error-free pattern and the tabulated single-fault patterns \( \{F_n(\theta_m, \phi_m)\}_{n \in C} \).

This will significantly improve the computational efficiency of the algorithm used in the search, for instance, the genetic algorithm above described. However, if the number of the array elements is moderate and the maximum number of faulty elements is restricted, an exhaustive search among all possible configurations of failed/unfailed elements may be feasible. In the present work, the maximum number of faulty elements is restricted to four. Although the methodology is directly applicable to higher number of faults, in real practice the probability of such large number of failures is small unless very large arrays are used.

### III. Application

In this section we have considered a linear array composed of \( N = 100 \) centre-fed cylindrical \( \lambda/2 \)-dipoles (of radius 0.005\( \lambda \), oriented parallel to the \( z \) axis with their centres placed along the \( x \)-axis with an inter-element distance of \( \lambda/2 \)). A ground plane is located at a distance of \( \lambda/4 \) behind the array (at \( y = -\lambda/4 \) as Fig. 1 shows).

With no failures, the dipoles are fed in order to radiate, in the \( \theta = 90^\circ \) plane, a pencil beam pattern with 1.2 deg. beamwidth measured at -3 dB and a sidelobe level (SLL) of -25 dB. The pattern, that is shown in Fig. 2, has been synthesized by sampling a linear Taylor distribution with \( SLL = -25 \) dB and \( n = 12 \), what yields the maximum efficiency for this particular sidelobe level [15]. When mutual coupling was taken into account, the impedance matrix was multiplied by the excitation column, obtained in the synthesis procedure, in order to calculate the initial voltages of the dipoles. This guarantees to start from the same error-free pattern in the coupling and non-coupling cases.

We simulated several configurations with 1 to 4 fully faulty elements randomly selected. In all the cases, a test set of 1250 patterns (12, 50, 150 and 1038 patterns with 1, 2, 3 and 4 faulty elements respectively) was used. This set, that contains \( M \) samples of each damaged pattern, is used as input of the algorithms to check their performance in the search. In our study, the test
patterns were sampled in the region $\phi \in [45^\circ, 135^\circ]$], in order to avoid extreme angles where the pattern may be difficult to measure. For simplicity, the samples were taken to be equispaced in each interval; however we have found that this is not strictly necessary.

In a first and ideal case, a total of $M = 50$ amplitude-only pattern samples with no measurement errors were considered. We studied the performance of several algorithms in the search: an exhaustive search (assuming a maximum number of faulty elements of 4, which yields a total of $\sum_{f=1}^{4} N_f \cdot [f(N - f)!] = 4,087,975$ different configurations to check) and also a genetic algorithm, both using the fast method described above. For comparison, the performance of a conventional genetic algorithm that evaluates the array factor for every chromosome/configuration (as in [11]) is also shown. Table I shows the percentage of the test set patterns in which the algorithm found the right solution (success rate), the average number of the patterns evaluated ($N_\text{ev}$ evaluations) and the average computer time for each test pattern, running on a desktop PC with a 2.4 GHz Core 2 Duo processor. Without considering mutual coupling (i.e., assuming current sources), expressions (1) and (4)—using the array factor and the single-fault patterns $F_{\text{ff}}$ respectively—give the same pattern, therefore an exhaustive search was able to find the faulty elements in whole test set as expected. On the other hand, although the conventional genetic algorithm [11] required a much lower number of evaluations, it is about 12 times slower than the exhaustive search. We also found that the use of a genetic algorithm based on the proposed method may be very useful when an exhaustive search is not feasible (if $N$ or the maximum number of failed elements is increased, the number of possible combinations boosts). Since the pattern considered in this example is generated by pure real symmetric excitations, the degraded far field pattern is the same for a given configuration of failed elements, say $C = \{f_1, f_2, \ldots, f_F\}$, and for its symmetric configuration with respect to the center of the array, $C_{\text{sim}} = \{N + 1 - f_1, N + 1 - f_2, \ldots, N + 1 - f_F\}$. Because of this, in the calculation of the success rate shown in the Table I for the no mutual coupling case, we considered that the algorithm found the right solution regardless of whether it provided $C$ or $C_{\text{sim}}$. This ambiguity in the solution disappears when mutual coupling is considered because, in this case, the currents of the dipoles near the faulty elements are not longer pure real.

Last three rows of the Table I show the performance of these algorithms when mutual coupling is considered (i.e., assuming voltage sources). In this case, the test set of 1250 patterns as described above: for each pattern, the impedance matrix is inverted to obtain the dipole currents that are finally used in (1) to compute the degraded pattern. This procedure, that is very slow, is the same as the used by the conventional genetic algorithm for every configuration, what guarantees the highest success rate.

### Table I

<table>
<thead>
<tr>
<th>Patterns without measurement errors</th>
<th>Proposed method</th>
<th>Conventional genetic algorithm [11]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exhaustive search</td>
<td>Genetic algorithm</td>
</tr>
<tr>
<td>No mutual coupling</td>
<td>Success rate</td>
<td>100 %</td>
</tr>
<tr>
<td></td>
<td>$N_\text{ev}$</td>
<td>4,087,975</td>
</tr>
<tr>
<td></td>
<td>Time</td>
<td>14.3 s</td>
</tr>
<tr>
<td>Mutual coupling</td>
<td>Success rate</td>
<td>98.8 %</td>
</tr>
<tr>
<td></td>
<td>$N_\text{ev}$</td>
<td>4,087,975</td>
</tr>
<tr>
<td></td>
<td>Time</td>
<td>14.3 s</td>
</tr>
</tbody>
</table>

Fig. 1. Geometry of the linear array of dipoles and a ground plane, together with the spherical and cartesian coordinates employed.

Fig. 2. Linear Taylor pattern radiated by the 100 element array of $\lambda/2$-dipoles with no failures in the $\theta = 90^\circ$ plane.
TABLE II
PERFORMANCE OF AN EXHAUSTIVE SEARCH WITH MUTUAL COUPLING USING AMPLITUDE-ONLY PATTERN SAMPLES. MEASUREMENT ERRORS WERE SIMULATED

<table>
<thead>
<tr>
<th>Number of amplitude-only pattern samples</th>
<th>$M=8$</th>
<th>$M=10$</th>
<th>$M=12$</th>
<th>$M=16$</th>
<th>$M=20$</th>
<th>$M=30$</th>
<th>$M=50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (exhaustive search)</td>
<td>2.3 s</td>
<td>2.8 s</td>
<td>3.4 s</td>
<td>4.5 s</td>
<td>5.7 s</td>
<td>8.6 s</td>
<td>14.3 s</td>
</tr>
<tr>
<td>Success Rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta_{\text{amp}}=0.5$ dB</td>
<td>74.2 %</td>
<td>87.0 %</td>
<td>90.1 %</td>
<td>91.8 %</td>
<td>94.8 %</td>
<td>97.6 %</td>
<td>98.7 %</td>
</tr>
<tr>
<td>$\Delta_{\text{amp}}=1.0$ dB</td>
<td>26.3 %</td>
<td>60.2 %</td>
<td>82.6 %</td>
<td>90.4 %</td>
<td>94.6 %</td>
<td>97.5 %</td>
<td>98.6 %</td>
</tr>
<tr>
<td>$\Delta_{\text{amp}}=1.5$ dB</td>
<td>9.5 %</td>
<td>26.6 %</td>
<td>53.4 %</td>
<td>84.6 %</td>
<td>93.6 %</td>
<td>97.2 %</td>
<td>98.6 %</td>
</tr>
<tr>
<td>$\Delta_{\text{amp}}=2.0$ dB</td>
<td>3.5 %</td>
<td>12.3 %</td>
<td>28.9 %</td>
<td>69.1 %</td>
<td>89.9 %</td>
<td>97.0 %</td>
<td>98.5 %</td>
</tr>
</tbody>
</table>

TABLE III
PERFORMANCE OF AN EXHAUSTIVE SEARCH WITH MUTUAL COUPLING USING COMPLEX (AMPLITUDE AND PHASE) PATTERN SAMPLES. MEASUREMENT ERRORS WERE SIMULATED

<table>
<thead>
<tr>
<th>Number of complex pattern samples</th>
<th>$M=8$</th>
<th>$M=10$</th>
<th>$M=14$</th>
<th>$M=20$</th>
<th>$M=30$</th>
<th>$M=50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (exhaustive search)</td>
<td>2.3 s</td>
<td>2.8 s</td>
<td>4.0 s</td>
<td>5.7 s</td>
<td>8.6 s</td>
<td>14.3 s</td>
</tr>
<tr>
<td>Success Rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta_{\text{amp}}=0.5$ dB, $\Delta_{\text{ph}}=5$ deg</td>
<td>95.1 %</td>
<td>98.2 %</td>
<td>99.9 %</td>
<td>100 %</td>
<td>100 %</td>
<td></td>
</tr>
<tr>
<td>$\Delta_{\text{amp}}=1.0$ dB, $\Delta_{\text{ph}}=10$ deg</td>
<td>86.2 %</td>
<td>96.8 %</td>
<td>99.8 %</td>
<td>99.9 %</td>
<td>100 %</td>
<td>100 %</td>
</tr>
<tr>
<td>$\Delta_{\text{amp}}=2.0$ dB, $\Delta_{\text{ph}}=10$ deg</td>
<td>63.4 %</td>
<td>91.0 %</td>
<td>99.7 %</td>
<td>99.8 %</td>
<td>99.9 %</td>
<td>100 %</td>
</tr>
<tr>
<td>$\Delta_{\text{amp}}=2.0$ dB, $\Delta_{\text{ph}}=20$ deg</td>
<td>30.2 %</td>
<td>64.3 %</td>
<td>96.1 %</td>
<td>99.5 %</td>
<td>99.6 %</td>
<td>100 %</td>
</tr>
</tbody>
</table>

as it is shown in the table. As expected, the proposed method does not perform as well here, but with an appropriate number of samples, its success rate is high enough requiring much less computer time.

In a second example, we have considered a real case by introducing several deviations to the samples in order to account for the noise and measurement errors affecting the power pattern measured. More specifically, we assumed a maximum experimental error of $\pm \Delta_{\text{amp}}$ (in dBs) and $\pm \Delta_{\text{ph}}$ (in degrees) in measuring the amplitude and phase of the pattern, respectively. This is equivalent to add to the exact value of each simulated sample of the test set, obtained using (1), a random value calculated between $-\Delta_{\text{amp}}$ and $\Delta_{\text{amp}}$ for the amplitude of each sample and between $-\Delta_{\text{ph}}$ and $\Delta_{\text{ph}}$ for its phase when applied. Taking the mutual coupling into account, we changed the number $M$ of samples to study its influence in the algorithm performance. Table II shows the results for an exhaustive search using amplitude-only pattern samples and modifying $\Delta_{\text{amp}}$ from 0.5 dB to 2.0 dB ($\Delta_{\text{ph}}$ is not applied here because the samples are real). As expected, noise robustness increases with the number of samples because the information of the degraded pattern is higher. In this case, depending of the noise level, the success rate may be above 90% using 12 real pattern samples only. Also, for all the noise levels, a success rate above 97% is guaranteed with 30 samples, wasting 8.6 seconds per pattern.

Depending of the measurement setup, it may be possible to reduce the number of required pattern samples and, at the same time, keep a high success rate at the expense of using complex (amplitude and phase) pattern samples of the far-field pattern. Obviously, phase pattern provides extra information that increases the algorithm robustness to noise. Table III shows the results for an exhaustive search using complex samples and modifying both $\Delta_{\text{amp}}$ and $\Delta_{\text{ph}}$. Note that with the lowest noise level, the success rate is above 95% using 8 complex pattern samples only. This is also guaranteed for all the noise levels with $M=14$ or more.

The method has also been applied to other linear Taylor parameter distributions with sidelobe levels ranging from $-15$ dB to $-30$ dB achieving results very similar to the previous ones. Finally, the algorithm has been tested with a flat-topped beam pattern with 10.5 deg. beamwidth measured at $-3$ dB, $+0.1$ dB of ripple level in shaped region and asymmetric side lobes of $-20$ and $-25$ dB, that was synthesized by means of the Orchard-Elliott method [16]. In this case, the results were appreciably better than the obtained with the pencil beam because the excitations required to synthesize the shaped pattern are complex, what produces more variability in the degraded pattern in presence of the failures.

IV. CONCLUSION

We have proposed a simple and fast method of finding faulty elements in antenna arrays using a small number of far field amplitude-only pattern samples. Although an exhaustive search has been performed in many examples, a genetic algorithm based on the proposed method may be very useful if the number of array elements and the number of faulty elements is very high. Using an equispaced ($d = \lambda/2$) linear array of 100 $\lambda/2$-dipoles, the method has proven to be efficient in a real case when both mutual coupling and noise/measurement errors are taken into account. The number of required samples that guarantees a high success rate in the array diagnostics can be significantly reduced at the expense of using complex pattern samples. Although we have considered that the defective elements fail completely, the technique proposed can, in principle, be extended to detect partial failures by including additional values for the status of each dipole as in [11]. The proposed method is directly applicable to planar arrays.
RODRÍGUEZ-GONZÁLEZ et al.: RAPID METHOD FOR FINDING FAULTY ELEMENTS IN ANTENNA ARRAYS

REFERENCES


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