

Sum Capacity Maximization-based Power Allocation for Cluster-wise Distributed MU-MIMO

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Abstract Cluster-wise distributed multi-user multiple-output multiple-input (MU-MIMO) is a promising solution for improving the transmission quality in the fifth generation (5G) advanced systems. When the clustering is introduced to reduce the huge computational complexity of large-scale MU-MIMO, the inter-cluster-interference (ICI) is produced, thereby reducing the sum capacity. Reasonable power allocation can effectively improve the system capacity while the total transmit power remains unchanged. Therefore, in order to make up for this loss, in this paper, the uplink/downlink power allocation is considered by taking into account the ICI to maximize the sum capacity. A new power allocation method is proposed, which is to maximize the sum capacity of the system under the cluster-wise total power constraint and cell-wise total power constraint, respectively, while satisfying the minimum user capacity requirement. Then, by computer simulation, the proposed method is compared to the user-wise equal power allocation method in terms of the achievable sum capacity.

Keywords Clustering, Distributed MU-MIMO, Power allocation, 5G advanced.

1. Introduction

Along with increasing mobile data traffic, the use of extremely high frequency band e.g. mmWave and the denser deployment of access points are demanded. However, the deployment of large-scale multi-user multiple-output multiple-input (MU-MIMO) with popular co-located antenna deployment causes signal blockage problem and prohibitively high computational complexity. To avoid these two serious problems while achieving the improved system performance, we considered a cluster-wise distributed MU-MIMO system, in which antennas are spatially distributed at different locations over the base station coverage area (cell), then users and antennas are grouped into several separated clusters to perform cluster-wise small-scale MU-MIMO in parallel to efficiently reduce the computational complexity [1]. In return, cluster cluster-wise MU-MIMO introduces the inter-cluster-interference (ICI), which results in the degradation of sum capacity of the system [2].

Certainly, there are many possible ways to mitigate the sum capacity degradation due to clustering, such as the power allocation, coordinated scheduling between clusters and so on. In this paper, we try to solve the problem from the aspect of resource allocation optimization. In other words, according to the channel state information (CSI) of users, the corresponding transmit power are allocated to

maximize the capacity of the system while keeping the total transmit power of uplink and also that of downlink to a multiple of the total number of users. Moreover, users need to be guaranteed the minimum required capacity for their quality of service (QoS) [3].

Considering the above, in this paper, we propose a sum capacity maximization-based power allocation in our considered cluster-wise distributed MU-MIMO with the consideration of ICI. And the minimum required user capacity constraint and transmit power constraint are assumed in both downlink and uplink. For the second condition of the power allocation problem, we consider two schemes, i.e., cluster-wise constraint to limit the total transmit power of each cluster according to the number of users in the cluster, and cell-wise constraint to limit the total transmit power for whole cell based on the totality number of users.

The rest of this paper is organized as follows. In Chapter 2, the cluster-wise distributed MU-MIMO is introduced. Accordingly, the achievable capacity of downlink and uplink are derived. In Chapter 3, we describe the sum capacity maximization-based power allocation and introduce the sequential quadratic programming (SQP) [4] method to solve such a nonlinear inequality constraints optimal problem. Then, in Chapter 4, we show the simulation results on the sum capacity with our proposed

power allocation and compare with equal power allocation. Finally, some conclusions and future studies are given in Chapter 5.

2. System model and problem statement

We consider a single-cell distributed MU-MIMO with base station (BS) processing communication system as shown in Fig. 1. Over the BS cell, U single-antenna users communicate with the BS through A separated distributed antennas (DAs). All the users and DAs are joint to form K exclusive user-DA clusters by K-means algorithm based on their locations [5]. Then, assuming the perfect channel state information (CSI) is known by both users and the BS, zero-forcing (ZF) based data transmission is utilized in both downlink and uplink in each cluster separately to eliminate the inter-user-interference (IUI).

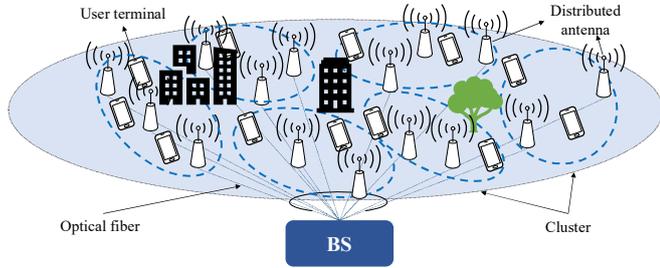


Fig. 1 Cluster-wise distributed MU-MIMO system.

The channel matrix and the ZF weight matrix for the k th cluster are represented by $\mathbf{H}_{k,k}$ and $\mathbf{W}_k = (\mathbf{H}_{k,k})^\dagger$ respectively, where $(\mathbf{A})^\dagger$ denotes the pseudo-inverse of matrix \mathbf{A} . Assuming that the power spectral density of user signal and that of noise have unity variance, we can derive the capacity for the u_k th user in the k th cluster as

$$C_{u_k}^\downarrow = \log_2 \left(1 + \frac{P_{u_k}}{\sum_{m=0, m \neq k}^{K-1} \sum_{j_m=0}^{U_m-1} \frac{P_{j_m}}{\|\mathbf{W}_m^\downarrow(:, j_m)\|^2} |\mathbf{h}_{u_k, m}^\downarrow \mathbf{W}_m^\downarrow(:, j_m)|^2 + 1}} \right) \quad (1)$$

$$C_{u_k}^\uparrow = \log_2 \left(1 + \frac{P_{u_k}}{\sum_{m=0, m \neq k}^{K-1} \sum_{j_m=0}^{U_m-1} P_{j_m} |\mathbf{W}_k^\uparrow(u_k, :) \mathbf{h}_{j_m, k}^\uparrow|^2 + \|\mathbf{W}_k^\uparrow(u_k, :)\|^2} \right)$$

In Eq. (1), P_{u_k} denotes the transmit power for the u_k th user in the k th cluster. $\mathbf{A}(x, :)$, $\mathbf{A}(:, x)$ and $\|\mathbf{A}\|$ denote the x th row vector, the x th column vector and the Frobenius norm of matrix \mathbf{A} , respectively.

Our objective is to maximize the sum capacity of whole system constrained by a total transmit power limitation and a minimum required user capacity. So, the optimization

problem is defined as

$$\begin{aligned} & \max_{P_{u_k}} \sum_{k=0}^{K-1} \sum_{u_k=0}^{U_k-1} C_{u_k} \quad (2) \\ & \text{s.t. } \mathbb{C}_1 : \sum_{k=0}^{K-1} \sum_{u_k=0}^{U_k-1} P_{u_k} = U \times P_{\text{target}} \\ & \quad \mathbb{C}_2 : C_{u_k} \geq C_{\text{required}}, \\ & \quad \forall k, u_k; k = \{0, 1, \dots, K-1\}, u_k = \{0, 1, \dots, U_k-1\} \end{aligned}$$

Here, for simplicity, we omit the superscript arrows representing the downlink and uplink, because the optimization problems are the same for these two conditions, but the calculation of the capacity part is different as indicated in Eq. (1).

We also realize that as long as the total transmit power of each cluster meets the limitation based on the number of users in the cluster, the total power of the whole cell also meets the limitation. Therefore, the condition 1 in above problem can be modified, as in Eq. (3).

$$\begin{aligned} & \max_{P_{u_k}} \sum_{k=0}^{K-1} \sum_{u_k=0}^{U_k-1} C_{u_k} \quad (3) \\ & \text{s.t. } \mathbb{C}_1 : \sum_{u_k=0}^{U_k-1} P_{u_k} = U_k \times P_{\text{target}}, \forall k = \{0, 1, \dots, K-1\} \\ & \quad \mathbb{C}_2 : C_{u_k} \geq C_{\text{required}}, \\ & \quad \forall k, u_k; k = \{0, 1, \dots, K-1\}, u_k = \{0, 1, \dots, U_k-1\} \end{aligned}$$

In order to clearly distinguish the two power constraints, we call the power constraint in Eq. (2) as cell-wise constraint. Corresponding to it, the power constraint in Eq. (3) is called cluster-wise constraint. The link capacities achievable by these two power constraints are compared later.

3. Sum capacity maximization-based power allocation by sequential quadratic programming

As we mentioned above, our objective sum capacity maximization problem with constrains are described in Eq. (2) and Eq. (3). To solve these two power allocation problems, we introduce Sequential Quadratic Programming (SQP) algorithm. Because it is a popular and robust methods for solving such a nonlinear constrained optimization. SQP is generally utilized to solve such a nonlinear inequality constrain programming problem, commonly expressed by

$$\begin{aligned} & \min_x f(x) \quad (4) \\ & \text{subject to } c_i(x) = 0, \quad i \in \mathcal{E} \\ & \quad c_i(x) \geq 0, \quad i \in \mathcal{I} \end{aligned}$$

The idea of SQP is to approximate the Hessian matrix of the original problem by iteratively solving the quadratic programming subproblem

$$\begin{aligned} & \min_d \nabla f(x_k)^T d + \frac{1}{2} d^T \mathbf{H}_k d \quad , \quad (5) \\ & \text{subject to } c_i(x_k) + \nabla c_i(x_k)^T d = 0, \quad i \in \mathcal{E} \\ & \quad \quad \quad c_i(x_k) + \nabla c_i(x_k)^T d \geq 0, \quad i \in \mathcal{I} \end{aligned}$$

by quasi-Newton updating method [6]. In Eq. (5), d is the search direction, k is the iteration index, \mathbf{H} is the Hessian matrix, and ∇ denotes the gradient. The concrete realization is as follows. Initialize a starting point x_0 , and approximation \mathbf{H}_0 . Then start the iteration, which mainly includes the following three steps: 1) Solve the subproblem in Eq. (5) to determine the search direction d_k ; 2) Determine the search step to update x_{k+1} ; 3) Update the Hessian matrix \mathbf{H}_{k+1} , until the stop condition.

4. Simulation results

In this Chapter, the considered sum capacity maximization problem is implemented by SQP method. Also, the two power constraint schemes and the equal power allocation case are compared by sum capacity. Here, we utilize the `fmincon`TM solver in MATLAB[®] throughout our simulation.

In simulation, we normalize the BS cell into a 1 by 1 square area, over which $U=64$ users and $A=128$ DAs are randomly located following uniform distribution, like shown in Fig. 2.

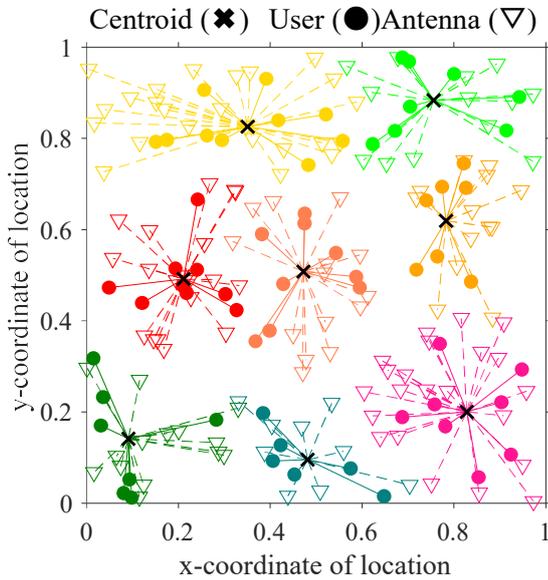


Fig. 2 An example of clustering result in the normalized BS cell, $K=8$

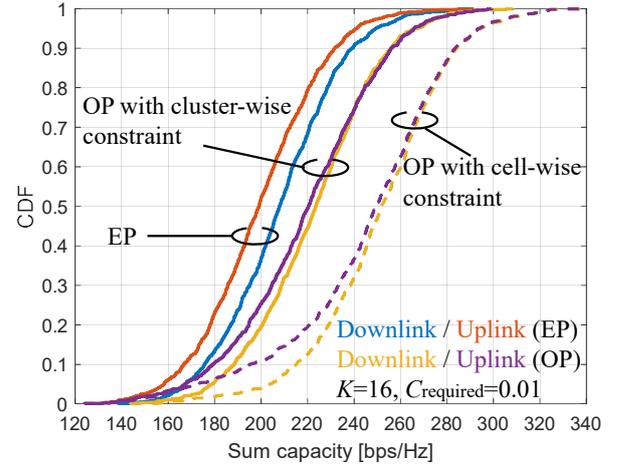
Throughout the simulation, the DA location remains unchanged. Different user locations are generated several times to calculate the cumulative distribution function (CDF) of sum capacity. The MIMO channel is characterized by distance-dependent path loss, log-normal

shadowing and frequency non-selective Rayleigh fading. The specific settings are shown in Table. 1.

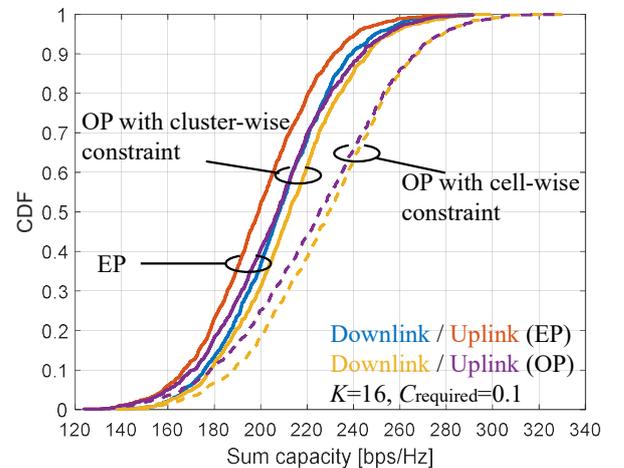
Table. 1

Number of DAs	128
Number of users	64
Number of clusters	4, 16, 64
Number of times of user location generations	1000
Path loss exponent	3.5
Shadowing standard deviation [dB]	8
Transmit power per user	1
Minimum required user capacity [bps/Hz]	0.01, 0.1, 0.5

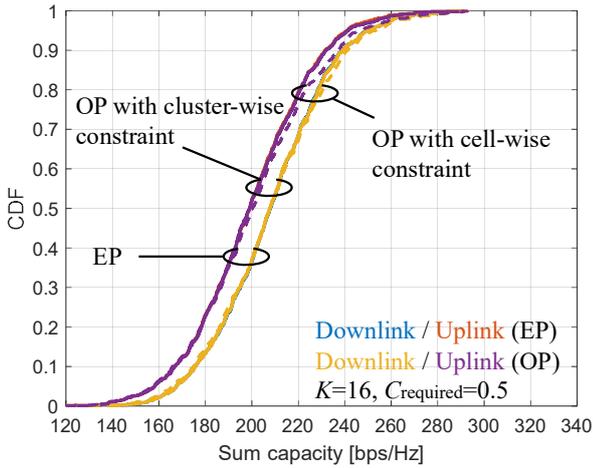
First, we plot the comparison of CDF of sum capacity for equal power (EP) allocation, optimal power (OP) allocation with cluster-wise constraint and cell-wise constraint in Fig. 3. It is worthy to note that if the optimization has no solution under such settings, the output sum capacity is equal to the EP case in this comparison.



(a) minimum required user capacity=0.01bps/Hz



(b) minimum required user capacity=0.1bps/Hz

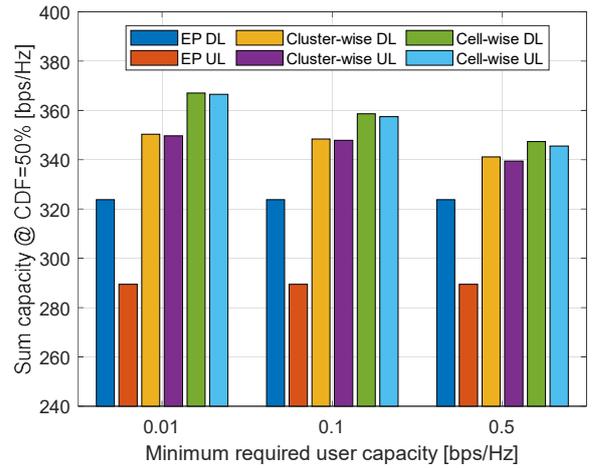


(c) minimum required user capacity=0.5bps/Hz

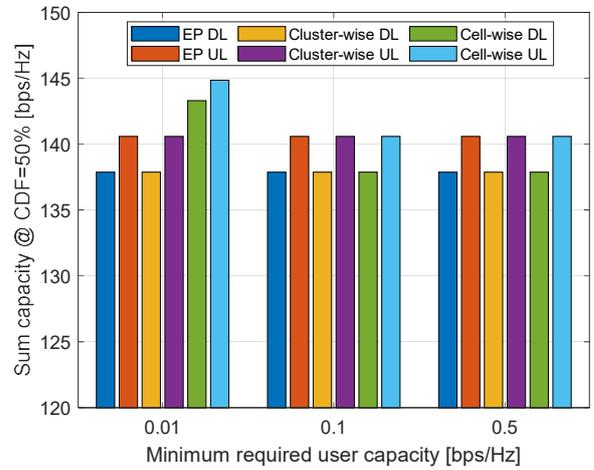
Fig. 3 CDF comparison of 3 power allocation schemes,
 $K=16$

As can be seen in the above figure, the proposed optimal power allocation can significantly improve the sum capacity in both downlink and uplink when the required user capacity is low. For example, when the required user capacity is set to 0.01bps/Hz, the sum capacity @ CDF=50% can be improved about 40bps/Hz and 20bps/Hz by cell-wise power constraint and cluster-wise constraint separately. As the minimum required user capacity increases, the improvement of sum capacity by power allocation gets smaller. As for the required user capacity is set to 0.5bps/Hz, the optimal solution can hardly be obtained under both power constraint schemes, but the cell-wise one is better than the cluster-wise one. According to Fig. 3 (c), when CDF is more than 50%, the curves of cluster-wise constraint and EP almost coincide, but cell-wise one is obviously different. This is because in fact cluster-wise constraint is only a solution set of cell-wise constraint, and its constraints are stricter, so it is not easy to get the optimal solution.

Without loss of generality, we further compare the three power allocation schemes in terms of sum capacity in a case of larger number of clusters ($K=4$) and in an extreme case ($K=U=64$) as shown in Fig. 4.



(a) $K=4$



(b) $K=U=64$

Fig. 4 CDF comparison of 3 power allocation schemes. The result of Fig. 4 is the same as that of Fig. 3, that is, cell-wise constraint is better, and the lower the minimum user capacity required, the easier the optimization is to get the optimal solution, and the larger the achievable capacity can be reached.

Combining with Fig. 3, we can see that when $K=4$, there is a significant improvement in the sum capacity for three different values of the minimum required user capacity irrespective of cluster-wise power constraint or cell-wise power constraint. However, as the number of clusters increases, the probability of getting the solution decreases and the advantage of our proposed power allocation diminishes. When $K=16$, solution when the minimum required user capacity is 0.5bps/Hz cannot be found. When $K=64$, it is found that the capacity of proposed method and that of EP are almost the same, in other words, there is almost always no solution to our proposed power allocation in case of the minimum required user capacity becomes 0.1bps/Hz. It can be said that our proposed power

allocation method is effective in the case of small number of clusters.

To sum up, the optimal power allocation method proposed in this paper improves the minimum user capacity and sum capacity in a variety of situations, among which cell-wise power constraint can be the best as a whole.

5. Conclusion

In this paper, we proposed a sum capacity maximization-based power allocation method based on SQP algorithm, under the total transmit power constraint and the minimum user capacity guarantee. Two types of total transmit power constraint were considered: cell-wise and cluster-wise. From Monte Carlo simulation, we found that the proposed sum capacity maximization-based power allocation method can greatly improve the sum capacity in the case of small number of clusters and/or lower minimum user capacity requirement. In addition, compared with cluster-wise transmit power constraint, there are more cases that cell-wise one can get the solution, and its achievable sum capacity is higher. Because of its more relaxed constraints, the feasible region is larger, in other words, the probability of getting the solution that satisfies the constraint becomes higher.

When the number of clusters increases, the proposed method often has no solution, unless the minimum power of the user is set to be very small, but this is meaningless in reality. Therefore, how to effectively increase the capacity when the number of clusters is large, for example, when $K=U$ as shown in simulation, will be left as our future study.

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