一般社団法人　電子情報通信学会
THE INSTITUTE OF ELECTRONICS,
INFORMATION AND COMMUNICATION ENGINEERS

信学技報
IEICE Technical Report
RCS2020-11(2020-05)

# A Study on Cluster-wise User-antenna Association in 5G Advanced Ultra-dense RAN

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**Abstract** Ultra-dense radio access network (RAN) consisting a large number of distributed antennas and user terminals is able to improve the capacity and coverage while saving transmission power in the 5G advanced systems. In order to alleviate computational complexity problem for large-scale multi-user multi-input multi-output (MU-MIMO) signal processing, user or antenna-based clustering is promising, in which a large-scale MU-MIMO is decomposed into many cluster-wise small-scale MU-MIMO. In each cluster, zero-forcing (ZF) precoding is utilized to eliminate inter-user interference (IUI). For the cluster-wise MU-MIMO, user-antenna association has a great impact on the link capacity. Previously, we proposed a distance-based user-antenna association method, but it has some flaws. In this paper, we propose other two methods to improve it. We adopt the well-known K-means clustering method. Two proposed methods are evaluated by achievable link capacity through computer simulation and are compared by heatmap in detail.

**Keywords** 5G advanced systems, Cluster-wise RAN, MU-MIMO, ZF, K-means, User-antenna association.

## 1. Introduction

With the popularization of all kinds of intelligent terminals, the aggregated mobile data traffic will increase explosively in 2020 and beyond. Reducing cell radius and increasing the number of low-power devices is one of the core technologies to ensure that the 5G advanced networks will support 1000 times traffic growth [1]. Distributed RAN is adopted to satisfy this point [2].

In our previous study, we applied the well-known K-means++ [3] clustering method to group the user terminals or antennas in the base station coverage area and proposed a user-antenna (U-A) association method to form cluster-wise MU-MIMO [6]. For this MU-MIMO downlink, we adopted ZF precoding in each cluster [4-5], so that the interference among users in each cluster can be eliminated perfectly in theory.

We found that the U-A association performed after the user clustering or antenna clustering strongly affects the performance of the link capacity of the system. So, in this paper, we are focusing on the U-A association. We found that the previously considered U-A association has some shortcomings and proposed two new methods to improve it.

The rest of this article is organized as follows. In Chapter 2, the distributed antenna ultra-dense MU-MIMO we consider in this paper is introduced. Then, the construction of cluster-wise MU-MIMO with clustering and U-A association is presented. At last, the downlink transmission channel model is presented. In the Chapter 3, U-A association methods are introduced in detail. In Chapter 4, two new association methods are evaluated by simulation of link capacity. User-clustering and antenna-clustering are also evaluated. Finally, in Chapter 5 offers some concluding remarks and our future research direction.

## 2. Cluster-wise distributed MU-MIMO

In this Chapter, the interest cluster-wise MU-MIMO RAN mentioned above is descripted in detail.

### 2.1 Construction of cluster-wise MU-MIMO

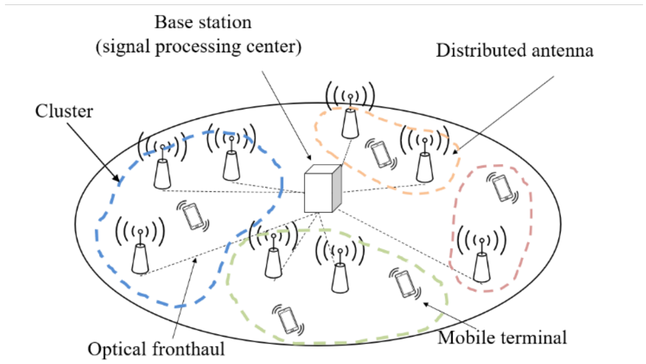



Fig. 1 Distributed MU-MIMO system with clustering.

Different from the centralized, distributed antenna deployment is that all antennas are scattered in the BS coverage area. Each distributed antenna is connected with BS via optical mobile fronthaul. A typical model of MU-

MIMO using distributed antennas is illustrated in Fig. 1.

All users in the BS coverage area simultaneously communicate with BS via all distributed antennas. The transmission quality and spectral efficiency can be greatly improved due to large diversity and/or multiplexing gain. Furthermore, compared with the traditional cellular systems, the radio link in the distributed antenna system is shorter, so the transmission power can also be effectively reduced, thereby improving the energy efficiency [6].

In our previous study [6], we proposed two types of clustering: *user-clustering* and *antenna-clustering* assuming that BS knows the location of all users and antennas in its coverage. After constructing user-clusters or antenna-clusters, U-A association is carried out to assign users (antennas) into user (antenna) clusters to build the cluster-wise MU-MIMO. The image of clusters is also indicated as the color dash lines in Fig. 1.

### 2.2 Downlink cluster-wise distributed MU-MIMO using ZF

We consider a cluster-wise distributed MU-MIMO with *A* distributed antennas and *U* users in one BS coverage, where each user is equipped with a single antenna as shown in Fig. 2. *N* clusters are formed by using K-means ++ algorithm and U-A association as we mentioned before.

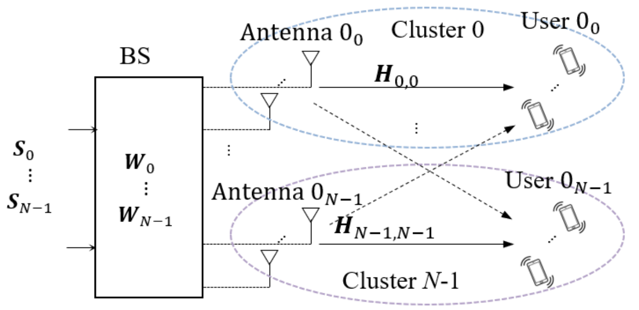



Fig. 2 Downlink transmission model of cluster-wise distributed MU-MIMO.

According to Fig. 2, the total pro-coded transmission signal $\boldsymbol{x}$ at antenna ports of *N* clusters can be expressed as

$$\boldsymbol{x} = \sqrt{\boldsymbol{P}}\boldsymbol{W}\boldsymbol{s}, \qquad (1)$$

where $\boldsymbol{x}$ is the $A \times 1$ transmission row vector, $\boldsymbol{s}$ is the $U \times 1$ signal row vector, $\boldsymbol{W}$ is the $A \times U$ precoding matrix, $\boldsymbol{P}$ is the $A \times A$ transmission power diagonal matrix. Then, the totality of received signal $\boldsymbol{y}$ at the user side can also be expressed as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} = \boldsymbol{H}\sqrt{\boldsymbol{P}}\boldsymbol{W}\boldsymbol{s} + \boldsymbol{n}, \qquad (2)$$

where $\boldsymbol{H}$ is the $U \times A$ downlink channel matrix, $\boldsymbol{n}$ is the $U \times 1$ noise row vector, also $\boldsymbol{y}$ has a size of $U \times 1$.

By representing the cluster index by $n \in \{0,1,\cdots,N-1\}$, and representing user index and antenna index in the *n*th cluster by $u_n \in \{0,1,\cdots,U_n-1\}$ and $a_n \in \{0,1,\cdots,A_n-1\}$. Aiming to form the cluster-wise MU-MIMO with ZF precoding, the number of users has to be less or equal to the number of antennas in each cluster, i.e., $U_n \le A_n$. The above matrices can be expressed more detailly as below.

$$\begin{cases} \boldsymbol{H} = [\boldsymbol{H}_0^T \quad \boldsymbol{H}_1^T \quad \cdots \quad \boldsymbol{H}_n^T \quad \cdots \quad \boldsymbol{H}_{N-1}^T]^T \\ \boldsymbol{H}_n = [\boldsymbol{H}_{n,0} \quad \boldsymbol{H}_{n,1} \quad \cdots \quad \boldsymbol{H}_{n,n} \quad \cdots \quad \boldsymbol{H}_{n,N-1}] \\ \boldsymbol{H}_{n,n} = \left[\boldsymbol{h}_{0_n,n}^T \quad \cdots \quad \boldsymbol{h}_{u_n,n}^T \quad \cdots \quad \boldsymbol{h}_{U_n-1,n}^T\right]^T \\ = \begin{bmatrix} h_{0_n,0_n} & \cdots & h_{0_n,A_n-1} \\ \vdots & \ddots & \vdots \\ h_{U_n-1,0_n} & \cdots & h_{U_n-1,A_n-1} \end{bmatrix} \end{cases}. \quad (3)$$

It is assumed that each cluster-wise channel $\boldsymbol{H}_{n,n}$ is perfectly known at BS. The ZF precoding matrix of each cluster $\boldsymbol{W}_n$ can be expressed as

$$\begin{cases} \boldsymbol{W} = diag[\boldsymbol{W}_0 \quad \boldsymbol{W}_1 \quad \cdots \quad \boldsymbol{W}_n \quad \cdots \quad \boldsymbol{W}_{N-1}] \\ \boldsymbol{W}_n = {\boldsymbol{H}_{n,n}}^{\dagger} = \boldsymbol{H}_{n,n}^H\left(\boldsymbol{H}_{n,n}\boldsymbol{H}_{n,n}^H\right)^{-1} \\ = [\boldsymbol{w}_{0_n} \quad \cdots \quad \boldsymbol{w}_{u_n} \quad \cdots \quad \boldsymbol{w}_{U_n-1}] \\ = \begin{bmatrix} w_{0_n,0_n} & \cdots & w_{u_n-1,0_n} \\ \vdots & \ddots & \vdots \\ w_{0_n,A_n-1} & \cdots & w_{U_n-1,A_n-1} \end{bmatrix} \end{cases}, \quad (4)$$

where $(\boldsymbol{M})^H$ represents the conjugate transpose of a matrix $\boldsymbol{M}$. In order to keep the transmit power per user at the same level $P$, ZF precoding matrix is normalized as shows in Eq. (5).

$$\begin{cases} \boldsymbol{P} = diag[\boldsymbol{P}_0 \quad \boldsymbol{P}_1 \quad \cdots \quad \boldsymbol{P}_n \quad \cdots \quad \boldsymbol{P}_{N-1}] \\ \boldsymbol{P}_n = \frac{U_n P}{\|\boldsymbol{W}_n\|_F^2} \times \boldsymbol{I}_{A_n} = diag[P_{0_n} \quad P_{1_n} \quad \cdots \quad P_{a_n} \quad \cdots \quad P_{A_n-1}] \end{cases}, \quad (5)$$

where $\boldsymbol{I}_{A_n}$ represents the $A_n \times A_n$ unit matrix.

Then, the received signal of the *u*th user in the *n*th cluster can be given as

$$y_{u_n} = \boldsymbol{h}_{u_n,n}^T\sqrt{P_n}\boldsymbol{w}_{u_n}s_{u_n} + \sum_{u_n'=0,u_n'\neq u_n}^{U_n-1}\boldsymbol{h}_{u_n,n}^T\sqrt{P_n}\boldsymbol{w}_{u_n'}s_{u_n'}$$

$$+\sum_{n'=0,n'\neq n}^{N-1}\sum_{u_{n'}'=0}^{U_{n'}-1}\boldsymbol{h}_{u_{n'}',n'}^T\sqrt{P_{n'}}\boldsymbol{w}_{u_{n'}'}s_{u_{n'}'} + n_{u_n}, \quad (6)$$

Here, the elements of modulated signal vector $\boldsymbol{s}$ and the noise vector $\boldsymbol{n}$ have i.i.d. complex-Gaussian entries with zero-mean and unity variance. From Eq. 6, the received signal-to-interference plus noise ratio (SINR) at the *u*th user in the *n*th cluster can be computed using

$$SINR_{u_n} = \frac{P_n\left\|\boldsymbol{h}_{u_n,n}^T\boldsymbol{w}_{u_n}\right\|^2}{\sum_{n'=0,n'\neq n}^{N-1}\sum_{u_{n'}'=0}^{U_{n'}-1}P_{n'}\left\|\boldsymbol{h}_{u_{n'}',n'}^T\boldsymbol{w}_{u_{n'}'}\right\|^2+1}. \quad (7)$$

The downlink capacity can also be computed by

$$\begin{cases} C_{u_n} = \log_2(1 + SINR_{u_n}) \\ C_{sum} = \sum_{n=0}^{N-1}\sum_{u=0}^{U_n-1}C_{u_n} \end{cases}. \quad (8)$$

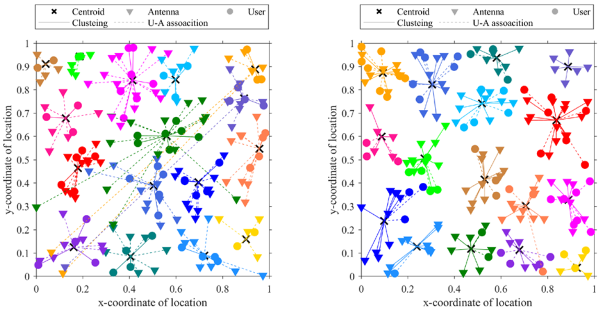
## 3. User-antenna association

In this Chapter, U-A association methods is introduced and discussed in detail. First, the previous U-A association algorithm is referred. Then two new methods are proposed to improve the previous one, and compared by some graphic samples.

### 3.1 Previously proposed U-A association

In [6], we adopted the shortest user-antenna distance-based U-A association. And in order to get more energy efficiency, we set all distributed antennas can be selected to service users.

Here a sample of the previous U-A association is shown in Fig. 3 with $A$=128, $U$=64, $N$=16.




(a) User-clustering (b) Antenna-clustering

Fig. 3 Result sample of previous U-A association.

In Fig. 3 (a), there are some users who choose the antennas far from their clusters. Because we use ZF in this article, and the number of antennas here is twice the number of users, due to the power allocation criteria of ZF, when the number of antennas near the user is sufficient, these extremely distant antennas can hardly allocate power. This will cause a waste of antenna resources.

On the other hand, although the antenna clustering case looks better from the graph, we found that the ratio of antenna and user in each cluster is uncertain instead of a uniform 1/2 in user-clustering. Even in some antenna distributions and user distributions, there may be no users in some antenna clusters. After our further investigation, this phenomenon becomes more obvious as the number of clusters increases. As in the example given in Fig. 3 (b), there is no user assigned to the purple antenna cluster in the upper right corner, which will also waste the antennas of this cluster.

For the reasons above, we want to improve this previous U-A association algorithm to reduce the waste of antenna resources and solve the problem of remote selections.

### 3.2 Proposed balanced U-A distance-based association

Considering that the previous algorithm [6] has a large difference between the standards of antenna-clustering and user-clustering, here we propose an improved algorithm that first uses a unified criterion to constrain the user-antenna ratio between two approaches, then assign the remaining users (antennas) by minimum distance. The specific expression is as Algorithm 1.

**Algorithm 1: Balanced U-A association**

**Input**: $U$ (user set), $A$ (antenna set)

**Output**: a set of user-antenna pairs

**User clustering case**:

1. Assign antennas to user clusters in ascending order of U-A distance. If $A_n = \left\lfloor U_n \cdot \frac{A}{U} \right\rfloor$ is satisfied.

2. Assign unassigned antennas to user clusters in ascending order of U-A distance. If $U_n \le A_n$.

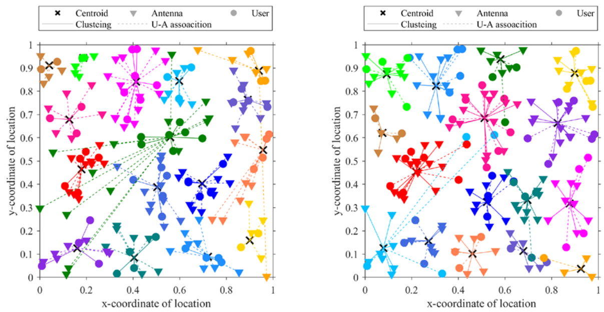
**Antenna clustering case**:

1. Assign users to antenna clusters in ascending order of U-A distance. If $U_n = \left\lfloor A_n \cdot \frac{U}{A} \right\rfloor (U_n \ge 1)$ is satisfied.

2. Assign unassigned users to antenna clusters in ascending order of U-A distance. If $U_n \le A_n$.

Here, $\lfloor \cdot \rfloor$ denotes the floor function that takes as input a real number and gives as output the greatest integer less than or equal to input number.

According to Algorithm 1 and maintaining the same antenna, user distribution and clustering results as in Fig. 3, we get the U-A association after clustering as follows.




(a) User-clustering (b) Antenna-clustering

Fig. 4 Result sample of Algorithm 1.

As shown in Fig. 4, since the same criteria are used this time, no matter the user clustering or antenna clustering, the user antenna ratio in each cluster can be kept almost the same as the ratio of the total number of users and antennas. In addition, due to our use of the floor function, the ratio of users to the total number of antennas can be any value, not limited to integers, which means this algorithm is more general.

However unfortunately, antennas or users that are far away from the clusters are still selected. Therefore, we consider whether we can reduce the occurrence of these long-distance selections, because the members in a cluster will reduce the capacity of the cluster at a distance, and it is more likely to cause interference to other clusters.

### 3.3 Proposed Cluster centroid-based U-A association

We found that if we want keep the U-A radio among clusters as mentioned in Algorithm 1, and the selection based on the user-antenna distance will inevitably lead to the appearance of long-distance selection. Therefore, we made the following two improvements: First, under the given conditions that can satisfy ZF ($U_n \leq A_n$), the number of users and antennas in each cluster is no longer restricted. Second, the U-A distance is no longer used, changed to the distance from the user (antenna) to the centroid of the antenna (user) cluster. How it works is described in Algorithm 2 and is illustrated as shown in Fig. 5.

**Algorithm 2: Distance to cluster's centroid-based U-A association**

**Input**: $U$ (user set), $A$ (antenna set)

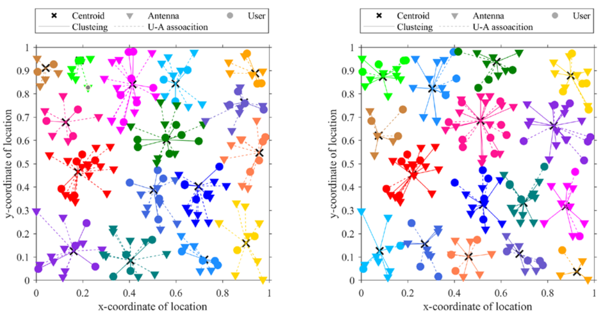
**Output**: a set of user-antenna pairs

**User clustering case**:

1. Assign antennas to user clusters in ascending order of A-C distance. Make sure $A_n = U_n$ is satisfied.
2. Assign unassigned antennas to user clusters in ascending order of A-C distance. If $U_n \leq A_n$.

**Antenna clustering case**:

1. Assign users to antenna clusters in ascending order of U-C distance. Make sure $U_n = 1$ is satisfied.
2. Assign unassigned users to antenna clusters in ascending order of U-C distance. If $U_n \leq A_n$.




(a) User-clustering (b) Antenna-clustering

Fig. 5 Result sample of Algorithm 2.

It can be seen from Fig. 5 that under the operation of Algorithm 2, the results of user clustering and antenna clustering become more compact, and the long-distance selection in Algorithm 1 no longer appears. It may increase the link capacity. However, on the contrary, the number of users and antennas in the clustering becomes uneven again. Since this algorithm is based on the distance of the centroids of the clusters, it can also be understood as one more clustering with the center position fixed after the first term clustering or 'double layer clustering'.

## 4. Comparison of two new U-A association methods by link capacity

In this Chapter, the two new U-A association methods are compared by the downlink capacity.

### 4.1 Simulation setting

The transmit power for each user is set so that the received signal-to-noise ratio becomes -20dB when the distance between the transmitter and receiver is equal to the side length of square-shaped BS area. The channel gain in Eq. (3) can be expressed as

$$h_{u_n,a_n} = \sqrt{{d_{u_n,a_n}}^{-\alpha}}\sqrt{10^{-\frac{\varphi_{dB}}{10}}}\, z, \quad (9)$$

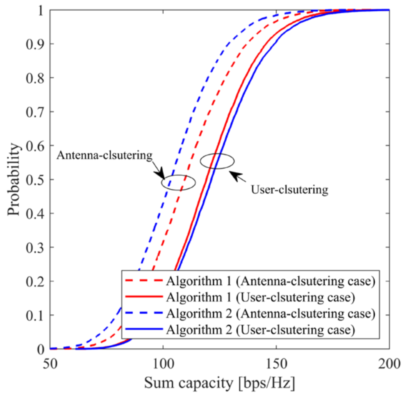
where $d_{u_n,a_n}$ is the distance between the $u$th user and the $a$th antenna in the $n$th cluster, $\alpha = 3.5$ is the pathloss exponent, $\varphi_{dB}$ is a zero-mean Gaussian random variable with standard deviation of $\sigma_{dB}$=8, and z is a complex-valued zero-mean Gaussian random variable with unit variance which represents Rayleigh fading.

For comparison, we fixed the antenna distribution as shown in Figs. 3-5, then randomly generated 1000 groups of user locations and do clustering and build U-A association once to form a sample of pathloss channel. Then 10 times shadowing for each pathloss and 10 times fading for each shadowing to get the samples of capacity. It should to be emphasized that we take a local average of capacity of 10 times fading to estimate user's service quality in a certain period of time. In other words, the CDF of sum capacity contains 1000×10×1=10000 samples.

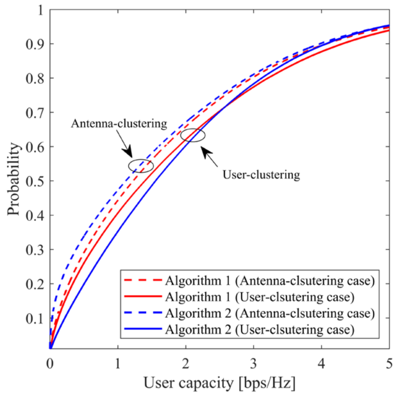
### 4.2 Comparison by CDF of link capacity

Due to we have two clustering approaches and two new association methods, there are four combinations. As shown in Fig. 6, it reveals that no matter which kind of U-A association method is adopted, user-clustering case achieves higher capacity. Then, separately observe from user-clustering case and antenna-clustering case to individually compare U-A association methods. In user-clustering case, Algorithm 2 gets a better result compared with Algorithm 1, but just a little. In antenna-clustering case, the result is opposite to the former. This result is very interesting.

Since Algorithm 2 is an algorithm that can be understood as a double-layer clustering, and the result of clustering is very dependent on the distribution of samples, we doubt whether there will be a different conclusion about the position of the antennas. We have tried several different antenna distributions and found that the comparison results are the same, which means that with enough user position sample calculations, the antenna position distribution does not affect the comparison results.




(a) CDF of sum capacity



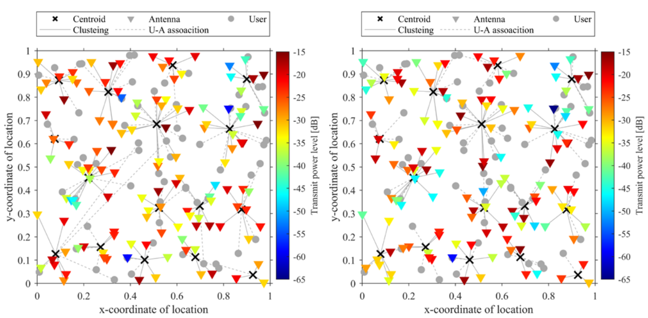

(b) CDF of user capacity

Fig. 6 Comparison of two clustering approaches and U-A association methods in CDF of link capacity.

Not as our previously imagining, making clustering more compact can increase capacity simply, and there may be other factors in the process. So, it is necessary to take a look, in detail, at how U-A association effects on link capacity.

### 4.3 Detailed comparison by heatmap

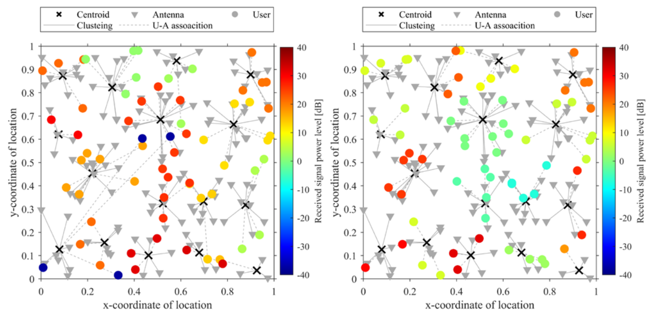
In this section, we take the same cluster results in Fig. 4 (b) and Fig. 5 (b) and observe by heatmap.




(a) Algorithm 1 (b) Algorithm 2

Fig. 7 Heatmap of transmit power level.

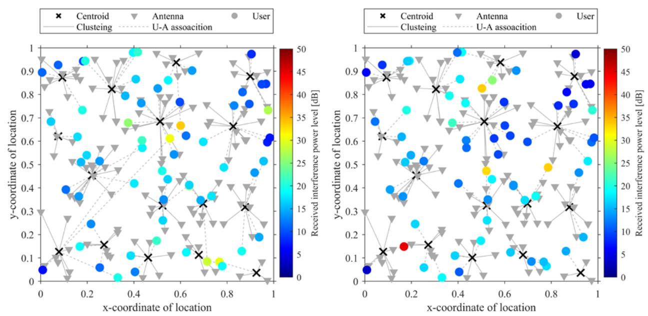
The power assignment of the antenna closest to each user (distance competition based) is mainly used to serve this user. Due to ZF is aiming to make every user got the same level of received signal power, the greater the distance of this invisible user antenna pair, the greater the power allocated to the antenna, and the lower the power of the remaining antennas. From Fig. 7, the antennas whose power lower than -45dB and marked as cyan and blue are more in the case of using Algorithm 2.




(a) Algorithm 1 (b) Algorithm 2

Fig. 8 Heatmap of signal power level.

Each user in the same cluster has the same signal power because of ZF precoding as shown in Fig. 8. Although the signal power of users in one cluster in (a) is extremely low, for other users, they are almost above 0dB, about half of users in (b) are below 0dB. This is consistent with the antenna power results.




(a) Algorithm 1 (b) Algorithm 2

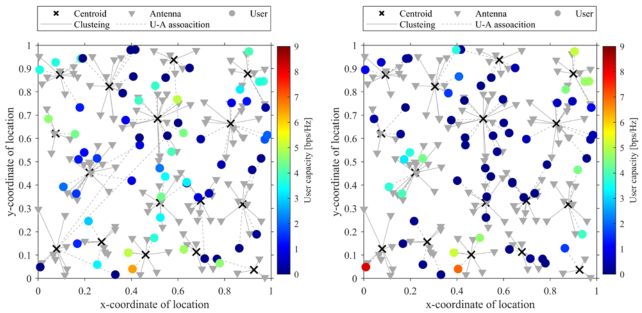
Fig. 9 Heatmap of interference power level.

Similarly, from Fig. 9, he overall level of interference power using the two methods is approximately the same. Combining Fig. 8 and Fig. 9, Algorithm 2 lead to a lower signal power distribution while keeping the interference constant which causes a lower link capacity. As for the reason, we consider that it should be related to the shortest user antenna pair in each cluster mentioned earlier. the maximum value of the shortest U-A distance in each cluster may mainly affect capacity.

As the data in Table 1, in same antenna cluster, if the max shortest U-A distance is longer, the poorer level of capacity the users get in the cluster as shown in Fig. 10. Except for cluster #4 and #6, all other clusters conform to this inverse proportional relationship, which may also involve the specific user antenna position and the total transmit power caused by the different number of users in the cluster.

Table 1 Max shortest U-A distance in clusters

| Cluster index | #1 | #2 | #3 | #4 |
|---|---|---|---|---|
| Algorithm 1 | 0.1181 | 0.1065 | 0.0520 | 0.0861 |
| Algorithm 2 | 0.0466 | 0.0980 | 0.0520 | 0.0760 |
| Cluster index | #5 | #6 | #7 | #8 |
| Algorithm 1 | 0.2713 | 0.1664 | 0.0415 | 0.0854 |
| Algorithm 2 | 0.1111 | 0.1178 | 0.0415 | 0.1253 |
| Cluster index | #9 | #10 | #11 | #12 |
| Algorithm 1 | 0.0469 | 0.6372 | 0.1005 | 0.0876 |
| Algorithm 2 | 0.0853 | 0.0482 | 0.0728 | 0.0947 |
| Cluster index | #13 | #14 | #15 | #16 |
| Algorithm 1 | 0.1403 | 0.0905 | 0.0363 | 0.0433 |
| Algorithm 2 | 0.0684 | 0.1214 | 0.1103 | 0.1563 |




(a) Algorithm 1 (b) Algorithm 2

Fig. 10 Heatmap of user capacity.

To draw a conclusion, Algorithm 2 suppresses the transmit power of the antenna to a certain extent, resulting in a reduction in signal power. Although the interference power is also reduced, the signal power has a greater attenuation level, resulting in a reduction in the overall channel capacity. Although Algorithm 2 makes each cluster more compact and eliminates the extreme case of long-distance selection, but the maximum value of the shortest distance of the user antenna in the cluster is generally increased compared to Algorithm 1. Applying Algorithm 1 may have extreme conditions, but the U-A shortest distance which effects the power allocation tends to be shorter. Namely, from Fig. 10, if the distribution of users selected by U-A association is close or overlapping to the antenna distribution, the capacity will be improved.

On the contrary, in user-clustering case, because of the number of users is less than the number of antennas, while clustering becomes compact, it also ensures that the minimum user antenna distance also becomes smaller. This is why Algorithm 2 performs better in user-clustering case.

## 5. Summary

In this paper, we focused on the U-A association of cluster-wise MU-MIMO. Because the previous method [6] has some problems, we proposed two new user-antenna association methods: the balanced U-A distance-based association method and to the cluster centroid-based association method. We made comparison of these two methods assuming two clustering approaches by CDF of link capacity. We found that no matter what U-A association method is used, user-clustering always performs better in terms of the link capacity because, in this paper, the number of users was assumed to be less than that of antennas.

Furthermore, we found that although clusters are formed compactly by applying Algorithm 2, the capacity does not necessarily increase; in the case of user-clustering the capacity increases but in the case of antenna-clustering the opposite result is obtained. So, from our results, the user-clustering with the centroid-based U-A association (Algorithm 2) can form compact clusters and achieve the highest link capacity.

## Acknowledgment

A part of this work was conducted under "R&D for further advancement of the 5th generation mobile communication system" commissioned by the Ministry of Internal Affairs and Communications in Japan.

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