

2-Steps Graph Coloring Algorithm for Interference Coordination in 5G Advanced Ultra-dense RAN

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Abstract The ultra-densification of radio access network (RAN) is considered as a promising approach to improve the coverage and the link capacity in 5G advanced systems. In ultra-dense RAN, the interference coordination is an important technical issue to efficiently utilize the limited radio bandwidth. The potential of graph coloring algorithm for interference coordination has already been verified by many researchers. This paper proposes a 2-steps graph coloring algorithm that can eliminate the inter-cell interferences and the inter-cluster interferences at the same time. In this paper, our proposed 2-step graph coloring algorithm is described. The first step is to apply the graph coloring algorithm on the cell edge in order to reduce the inter-cell interferences. Once the color of the cell edge has been decided, the second step is to utilize the conditioned graph coloring on the clusters within each cell. As a preliminary study, this paper is mainly focusing on the analysis of the second step. The computer simulation results show that the second step of our proposed 2-step graph coloring algorithm can improve the link capacity within each cell compared with no coloring case.

Keywords 2-step graph coloring, interference coordination, 5G advanced, ultra-dense RAN

1. Introduction

Nowadays, many countries have already started their 5G services [1]. Because of the shortage of the available radio bandwidth, millimeter wave has been used. Due to the nature of the rectilinear propagation of millimeter waves, radio link blockage may happen frequently [2]. In order to improve the coverage and the link capacity, the ultra-densification of radio access network (RAN) with distributed antennas is considered as a promising approach [3]. To further improve the spectrum efficiency, the multi-user multi-input multi-output (MU-MIMO) has been considered as a promising approach [4]. However, one disadvantage of a large-scale MU-MIMO is that the computational complexity is extremely high, therefore finding the computationally efficient solution towards ultra-dense RAN with distributed antennas is facing an urgent demand [5]. Our proposed solution is to group nearby users and antennas into clusters in each cell. With clusters, the large-scale MU-MIMO can be divided into several small-scale MU-MIMOs and the computational complexity will be reduced significantly. Besides that, same frequency will be reused in clusters, so the spectrum efficiency will be improved. Also, since the distance is relatively short in each cluster, the transmit power will be reduced, which in turn will result in the improvement in

energy efficiency. The disadvantage for introducing clusters into ultra-dense RAN is about the inter-cluster interference, so the method that can eliminate the inter-cluster interference is urgently required.

In this paper, graph coloring theory [6] is applied to eliminate the inter-cluster interferences. We creatively introduced the Delaunay Triangulation [7] to decide the neighboring relationship from the view of computational geometry and also, we proposed a restricted color number algorithm to control the maximum color number in order to make full use of the entire bandwidth. In this paper, clusters are formed by our proposed 2-layer clustering algorithm based on K-means++ algorithm [8]. Based on the computer simulation results of the sum and user link capacity, the effectiveness of our proposed graph coloring algorithm in single cell is verified. In our future study, this algorithm will be used as the foundation of our proposed 2-step graph coloring algorithm in a multicell system.

The rest of paper is organized as follows. In Chapter 2, our clustering algorithm and graph coloring algorithm is introduced. In Chapter 3, the results of graph coloring are presented and the link capacity is evaluated by computer simulation. Finally, Chapter 4 is the conclusion and future research plan.

2. MU-MIMO in ultra-dense RAN with

distributed antennas

2.1. Proposed clustering algorithm

In MU-MIMO with distributed antennas system, the antennas are spatially distributed instead of centralized in the BS coverage area. Through this way, the transmission power and spectral efficiency can be improved significantly. However, the computational complexity is also increased. In our previous research, forming clusters has been proved to be an efficient way to reduce the complexity while keep the link capacity to a comparable high level. In this paper, a new clustering algorithm named 2-layer clustering algorithm is proposed.

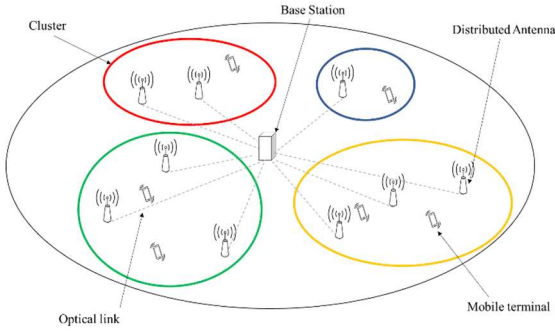


Fig.1. MU-MIMO using distributed antennas with clusters.

For computer simulation, we assume a normalized square-shaped BS area of 1 by 1, in which 128 distributed antennas and 32~128 users equipped with single antennas are randomly distributed. In our Monte Carlo simulation in Chap. 3, 128 antennas are randomly located and remain unchanged during all trials, while the user locations are changed every trial.

In Step 1, the distributed antennas will be grouped into several clusters based on K-means++ algorithm [9] according to the Euclidean distance. After that, the first layer of antenna clusters is formed and the cluster centroids is determined. Fig.2 illustrates the cluster layout composed of 8 antenna clusters.

In Step 2, based on the centroid, one-time K-means++ iteration is applied to form the second layer of user clusters (shown in Fig.3). With this method, we can guarantee to maximize the overlap region between the antenna clusters and user clusters.

The flow chart of the proposed 2-layer clustering algorithm is shown in Fig.4. Because of Zero-Forcing (ZF) precoding, the number of users must be equal to or smaller than the number of antennas in each cluster. In order to fulfill this requirement, we also introduce a trading algorithm to reorganize the number of users and antennas in each cluster. The details of our trading algorithm are

described as Step 3 (selection process) and Step 4 (reassignment process) in flow chart.

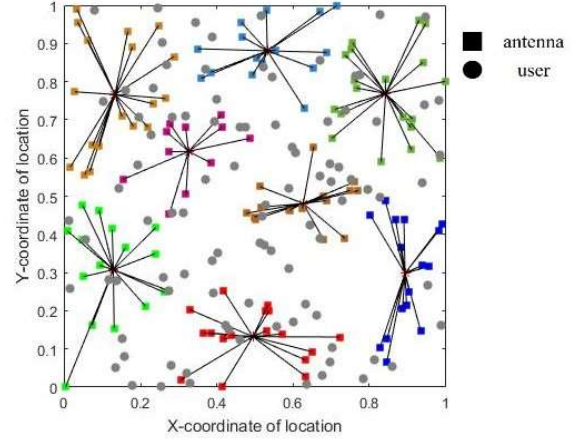


Fig.2. Antenna clusters formed in Step 1.

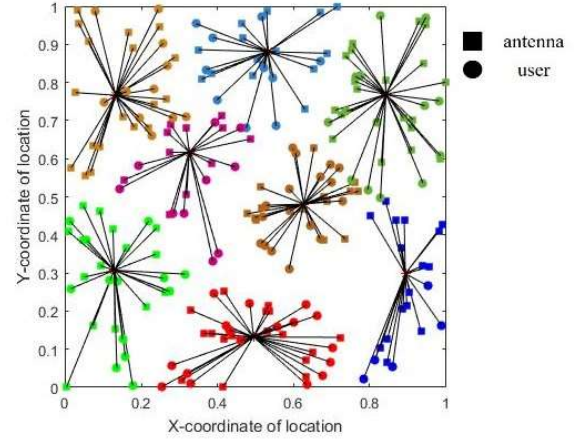


Fig.3. User clusters formed in Step 2.

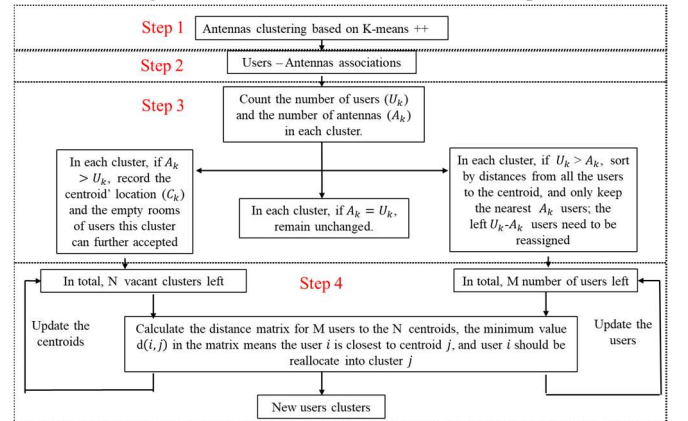


Fig.4. Flowchart of the proposed 2-layer clustering algorithm.

2.2. Proposed graph coloring algorithm

Graph coloring remains to be an unsolved problem in graph theory. The graph coloring problem in our RAN system can be abstracted into the simplest vertex coloring problem, where the clusters' location is abstracted into an undirected graph $G = (V, E)$, and V denotes vertices (centroid of clusters) while E denotes edges (neighboring

relationship). In its simplest form, it is a way of coloring the vertices of a graph such that no two adjacent vertices are of the same color [10]. For practical engineering application in wireless communication system, ‘colors’ means ‘frequency’, in short, our idea is to assign different frequency to neighboring clusters. Since the neighboring clusters do not share the same frequency, the inter-clusters interference will be eliminated. After graph coloring, all the clusters will be divided into different frequency groups, and in each group the clusters are not adjacent to each other. In order to represent the neighboring relationship, we define an adjacency matrix \mathbf{A} as

$$\mathbf{A} = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1N} \\ A_{21} & A_{22} & \cdots & A_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & \cdots & A_{NN} \end{pmatrix}, \quad (1)$$

where $A_{ij}=1$ indicates that the cluster i and cluster j are neighbors; otherwise, $A_{ij}=0$ indicates that the cluster i and cluster j are not adjacency to each other. In our proposed algorithm, the neighboring relationship among the clusters in each cell should be decided in advance based on Delaunay Triangulation (Subsection 2.2.1). After neighboring relationship has been decided, our proposed graph coloring algorithm is employed based on Backtracking schema. The complete Restricted Color Number algorithm will be explained in Subsection 2.2.2

2.2.1 Delaunay Triangulation

Delaunay Triangulation is a classical triangulation algorithm in computational geometry [12]. A Delaunay Triangulation for a given set of discrete points in a plane follows the restrictions that no point is inside the circumcircle of any triangles. Delaunay triangulations maximize the minimum angle in the triangles and it tends to avoid sliver triangles. In our algorithm, Delaunay Triangulation is the way to determine the adjacency relationship, or adjacency matrix \mathbf{A} , between the clusters. Fig.5 is the Delaunay Triangulation results for the distribution example in Fig.3, the vertexes of each triangles represent the centroid of clusters. If there is a line connecting two vertexes, these two vertexes will be regarded as neighbors and cannot share the same color, at the meantime, A_{ij} and A_{ji} equals 1 in adjacency matrix \mathbf{A} . The introduction of Delaunay Triangulation can scientifically decide the relative position relationship among all the clusters from the view of graphics and avoid personal judgment errors or deviations.

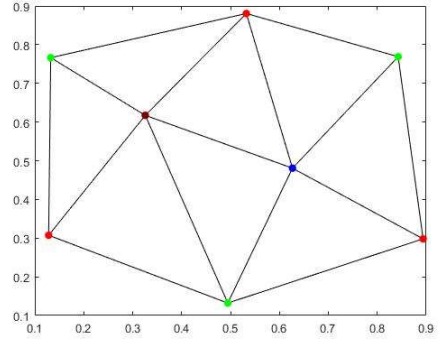


Fig.5. Delaunay Triangulation and graph coloring result.

2.2.2 Proposed Restricted Color Number algorithm

The essence of our proposed graph coloring algorithm is the restriction of maximum color number. With this restriction, we can avoid dividing the entire bandwidth into too many parts, also make sure that in each color group there are enough clusters to make full use of the bandwidth. The Restricted Color Number Algorithm consists of 3 steps:

- Step 1. Set the maximum color number.
- Step 2. Obtain adjacency matrix based on Delaunay Triangulation and then utilize graph coloring via Backtracking schema.
- Step 3. If the needed color number is larger than the maximum color number, choose one cluster nearby with the least interference and have its color.

The critical point is to find the least interfered cluster in Step 3. Relative distance in Eq. (2) is the parameter we put forward that can evaluate the inter-cluster interferences. The relative distance D_{ij} indicates the interference level from cluster j to cluster i if we view cluster as a whole and neglect the users and antennas locations inside the clusters. Then the task for searching the least interfered clusters turn into the task searching for the least value in each row of the relative distance matrix \mathbf{D} .

Cluster i 's relative distance is defined as

$$D_{ij} = \frac{d_{ij}^{-\gamma}}{\sum_{j=1, j \neq i}^n d_{ij}^{-\gamma}}, \quad j = 1 \sim N, \quad (2)$$

where d_{ij} denotes the distance from the cluster j 's centroid to cluster i 's centroid, N is the number of clusters, and γ is the pathloss exponent.

2.3. Cluster-wise MU-MIMO using ZF

In our previous research [9], we have already discussed about the link capacity calculation method in each cell

when clusters are introduced. When the graph coloring algorithm is applied, clusters are divided into several color groups, and each color group only has $1/CLR$ of the entire bandwidth. The downlink sum capacity can be computed using

$$C_{sum} = \frac{1}{CLR} \sum_{clr=0}^{CLR-1} \sum_{n_{clr}=0}^{N_{clr}-1} \sum_{u_{clr}=0}^{U_{n_{clr}}-1} \log_2(1 + SINR_{u_{n_{clr}}}), \quad (3)$$

where CLR is the maximum number of colors and $clr \in \{0, 1, \dots, CLR - 1\}$

3. Monte Carlo simulation & discussion

In this chapter, we evaluate the downlink sum capacity and the user capacity of cluster-wise MIMO using ZF. The normalized transmit signal power-to-noise ratio for each user is set to 0dB. The pathloss exponent is set to be 3.5, and the shadowing standard deviation is 8dB. The fading type chose frequency- nonselective Rayleigh fading. All the CDF results are based on 1,000,000 times trials for the certain set of antenna locations in Fig.2; the user locations changed every trial.

3.1. Examples of Coloring results

Figs.6-8 are the graph coloring results for the example shown in Fig.3 when the user number is 96 and the cluster number is 8. Based on our proposed Restricted Color Number Algorithm, we restricted the maximum color number to be 2, 3, or 4, and the sum capacity for this special circumstance is 92.7, 136.2 and 138.6 bps/Hz. As we mentioned before, 4 color can completely eliminate the inter-cluster interferences from the neighboring clusters, but at the meantime, each color group only get 1/4 of the bandwidth, and the increase in sum link capacity is not linear as the number of colors increase. This is the reason why we have to restrict the maximum color number.

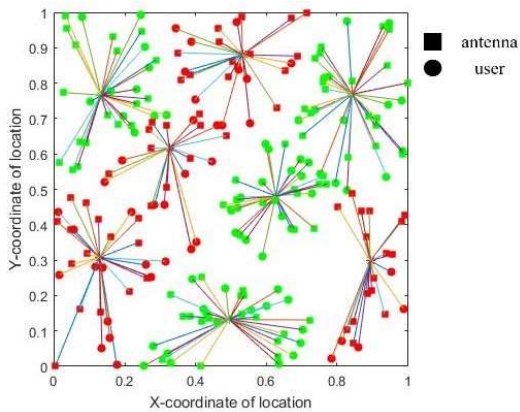


Fig.6. Graph coloring results when CLR=2.

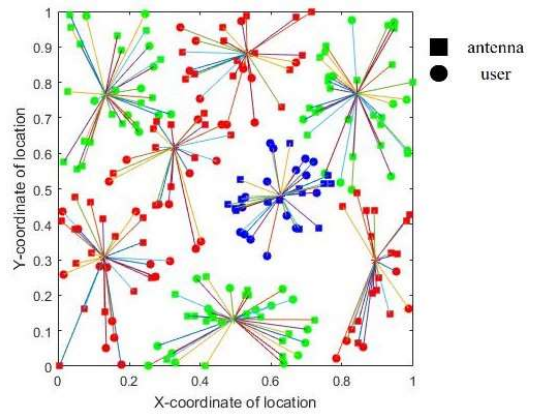


Fig.7. Graph coloring results when CLR=3.

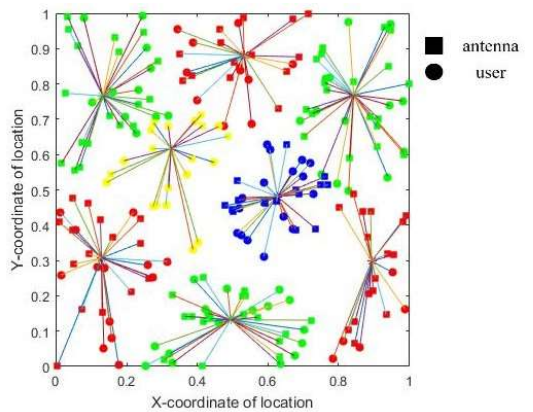
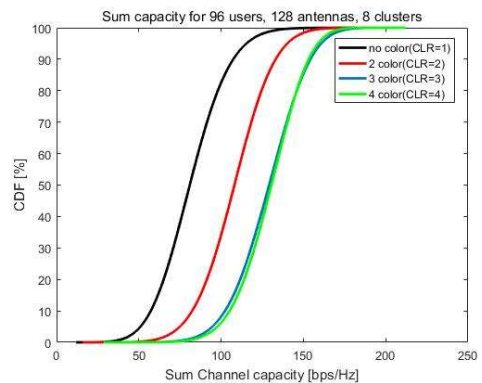


Fig.8. Graph coloring results when CLR=4.

3.2. Discussion of sum link capacity and user capacity

Fig.9 plots the cumulative distribution functions (CDFs) of the sum capacity and the user capacity for the case of 96 users, 128 antennas, and 8 clusters. The results reveal that overall, the graph coloring using 4 colors (i.e., CLR=4) provides the highest capacity, followed by using 3 and 2 colors (i.e., CLR=3 and 2), but the advantage is narrowing for graph coloring using 3 and 4 colors. No color (i.e., CLR=1) which means all clusters reuse the same whole bandwidth provides the lowest capacity.



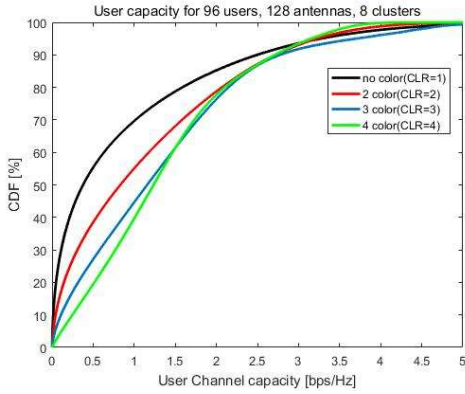


Fig.9. CDF of sum capacity and user capacity for 96 users, 128 antennas, and 8 clusters.

Fig.10 plots the 50% sum link capacity as a function of the number of clusters when the number of users is 32, 64, 96 and 128. The restricted maximum color number (CLR) is from 2 to 5 and the number of clusters is from 3 to 32. The simulation results reveal when the number of users is approaching to the number of antennas, the graph coloring algorithm performances better. When the number of users is 32, no coloring result is the best. When the number of users increases to 64, graph coloring algorithm performs better than the no coloring result when the number of clusters is less than 10. When the number of users increase to 94, the advantages of graph coloring extends to all the cluster numbers, and when the number of users is equal to the number of antennas (128), the advantages is more obvious. Since this is a preliminary research, we draw the rough conclusion that when the user number is above the half of the antennas' number, our proposed Restricted Color Number Algorithm have advantages in promoting the sum capacity.

Fig.11 compares the user capacity when the number of clusters equals 5, 10 and 15. In each comparison, the number of users is set between the number of clusters (at least each cluster has one user) and the number of antennas (to meet the Zero-Forcing's requirement). The results show our proposed graph coloring algorithm can improve the 10% user capacity for all cases of user number. Also, the 50% and 90% user capacity can be improved when the user number is more than half of the antennas number

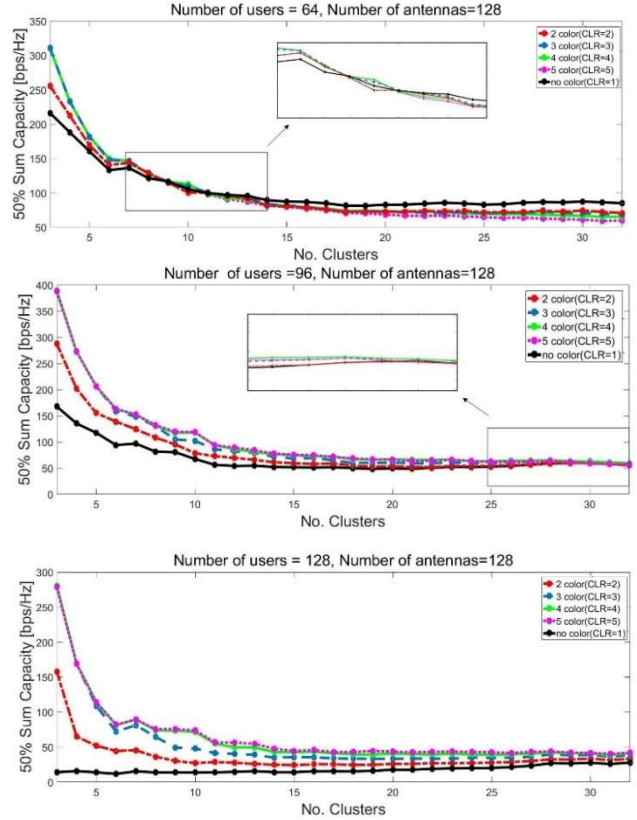


Fig.10. 50% sum link capacity as a function of the number of clusters when the number of users is 32, 64, 96 and 128.

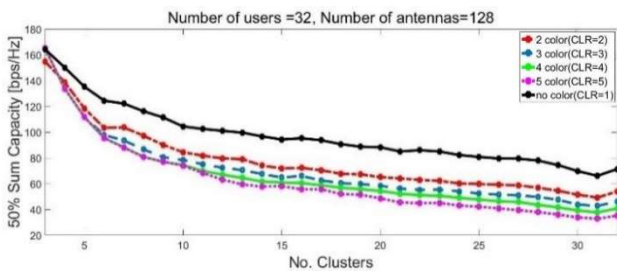
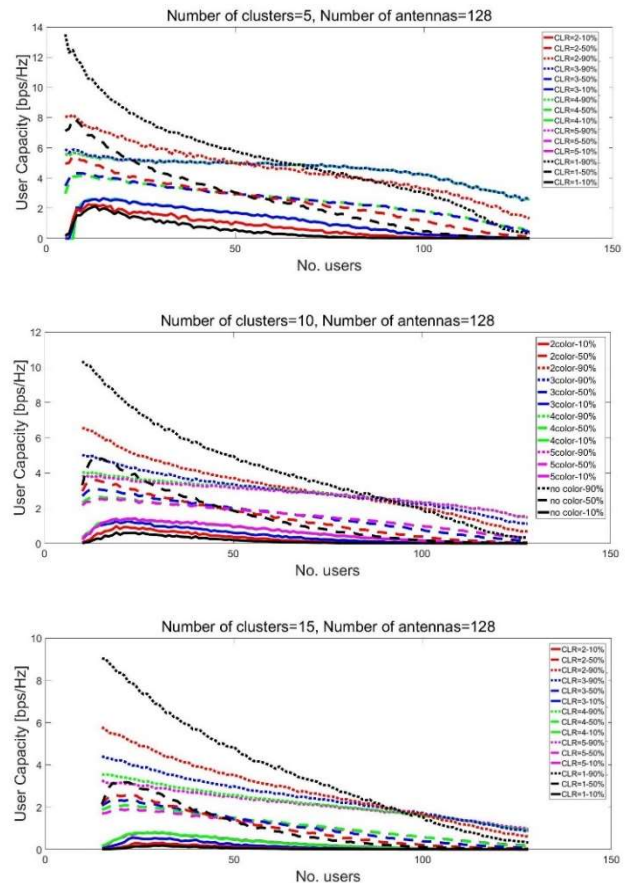


Fig.11. Comparison of user capacity when the number of clusters is 5, 10 and 15.

4. Conclusion

In this paper, we proposed a 2-layer clustering algorithm based on k-means ++, with this clustering algorithm, we can maximum the overlapping area between the user clusters and antenna clusters. Based on the clusters generated by this algorithm, we also proposed a restricted color number algorithm to eliminate the inter-cluster interference. In this algorithm, we creatively introduce Delaunay triangulation from computer graphics to decide the neighboring relationship from the view of graphics. Also, our proposed algorithm can restrict the maximum color number to fully utilize the entire bandwidth.

From our analysis, we have the following findings:

- i. From the sum capacity point of view, we compared the 50% sum capacity, and concluded that the advantage of graph coloring continues increasing when the number of users is approaching the number of antennas. In practical engineering application, we recommend to apply the graph coloring algorithm when the number of users is more than half of the number of antennas.
- ii. For users' side, the 50% and 90% user capacity also be improved when the number of users is more than half of the number of antennas, but the 10% user capacity performed well in all the situations. So, we can draw the conclusion that our proposed graph coloring algorithm can further enhance the users' experience and satisfaction.

For the future research, our next step is to introduce the restricted color number algorithm into multicell system and form a 2-step graph coloring algorithm. The first step is to color the cell edge to eliminate the inter-cell interference, and then inside each cell, our proposed restricted color number algorithm will be applied to eliminate the inter-cluster interference. After that, we also plan to introduce machine learning into our algorithm for further optimization.

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