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Fast MoM Analysis of Large-Scale Reflectarray by using Gauss-Seidel Scheme

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Abstract In this paper, the method of moments (MoM) combined with Gauss-Seidel method is applied to the numerical analysis of a large-scale array antenna with non-uniform sized elements. Numerical analysis shows that the CPU time of the Gauss-Seidel scheme for solving a matrix equation is proportional to the N^2, where N is the number of the array antenna's elements. This method can be extended easily for the numerical analysis of a reflectarray.

Keywords Method of Moments, Gauss-Seidel method, large-scale, non-uniform sized

1. Introduction

Nowadays, the design of a reflectarray, which improves a propagation channel, has much attention.[1] Especially, the design of a large-scale reflectarray is of great interests. However, the design of the large-scale reflectarray is difficult because long CPU time is required for its numerical analysis. On the other hand, it is well known that the method of moments (MoM) is one of the efficient methods for numerical analysis of antennas or scatterers. In the MoM, a N_T x N_T matrix equation is solved and the unknown current vector is obtained, where N_T is the number of unknowns. Because the CPU time for solving matrix equation is dominant in the MoM, how to solve the matrix equation fast is one of the attractive research topics.

Direct solvers are popular techniques for solving the matrix equation. For example, the CPU time of the Gauss-Jordan method, which is one of the most popular direct methods, is proportional to N_T^3. Therefore, when the scale of the matrix equation becomes very large, the CPU time increases greatly. The other problem is that direct methods will lead to enlarging the round-off error [2].

On the other hand, iterative solvers such as Conjugate Gradient method and Gauss-Seidel method have been proposed [3][4]. In these iterative methods, matrix-vector multiplication is carried out and the unknown current vector is updated iteratively. Because the CPU time for matrix-vector multiplication is proportional to N_T^2 and dominant, matrix-vector multiplication has been accelerated using various techniques, such as the fast multipole method (FMM) [5], multilevel fast multipole algorithm (MLFMA) [6], [7], and the fast inhomogeneous plane wave algorithm [8]. Due to these techniques, the total CPU time of the iterative method becomes much smaller than N_T^3 when the number of iterations is small. However, it is well-known that the number of iterations strongly depends on the problems to be solved.

In our previous study [9], the MoM combined with Gauss-Seidel method was employed for the numerical analysis of large-scale array antennas, which consist of array elements of uniform size. However, the MoM combined with Gauss-Seidel method has not been applied for numerical analysis of large-scale array antennas which consists of array elements of non-uniform size. In this paper, the MoM combined with the Gauss-Seidel method is employed for numerical analysis of the large-scale array antenna with non-uniform sized elements. The convergence criterion of this algorithm is investigated and the effectiveness of the method is shown numerically. This method can be easily applied for numerical analysis of a large-scale reflectarray.

2. Gauss-Seidel scheme

The important procedure for solving the matrix equation *[Z][I]=[V]* for unknown *[I]* by using the Gauss-Seidel scheme is to split the matrix *[Z]* into *[S]* and *[T]* so that the matrix equation becomes

$$[S][I] = -[T][I] + [V]. \tag{1}$$

where *[S]* contains the lower-left triangular part including the diagonal elements of *[Z]*, and *[T]* contains the upper-right triangular part excluding the diagonal elements. The iterative scheme for solving Eq. (1) is given by:

$$I_i^{(l_s+1)} = \frac{1}{S_{ii}}\left[V_i - \left(\sum_{j=1}^{i-1} S_{ij} I_j^{(l_s+1)} + \sum_{j=i+1}^{N} T_{ij} I_j^{(l_s)}\right)\right],$$

$$i = 1 \sim N, l_s = 1 \sim L_S. \quad (2)$$

where I_i, S_{ij} and T_{ij} are the elements of the vector *[I]*, matrices *[S]* and *[T]* respectively. The superscript *ls* is the step number of the iteration. The initial $I_i^{(0)}$ is usually assumed to be zero. This iteration continues until

$$\left|\frac{I_i^{(ls+1)} - I_i^{(ls)}}{I_i}\right| \le e_t\,, \quad (3)$$

for all i at the final L_S th step, where I_i indicates the ideal value of current and e_t indicates tolerance of relative error. The convergence criterion for the Gauss-Seidel scheme is that all the eigenvalues of the matrix $[S]^{-1}[T]$ have magnitudes less than unity [10].

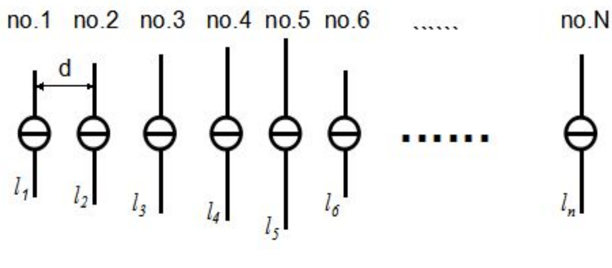

Fig.1 Analysis model: large-scale array antenna with non-uniform sized elements

The analysis model of the large-scale array antenna with non-uniform sized elements is shown in Fig. 1. The large-scale array antenna with non-uniform sized elements consists of a lot of dipole elements with different length. *N* is the total number of dipole elements. *d* is the space between two adjacent dipole elements, and is given a constant value (usually 0.5 λ , where λ is the wavelength) . Each dipole element has one segment. The length of *k* th dipole is defined as:

$$l_k = (0.45 + 0.025 \times \mathrm{mod}(k,5))(\lambda), \quad (4)$$

3. ANALYSIS OF CONVERGENCE

Figure 2 shows whether the iteration method can be applied or not. The value of max[$|I_i^{(ls+1)}-I_i^{(ls)}|$ / $|I_i|$] is calculated when iterative steps increase, which is also called the relative error. In this case, $d=\lambda/2$, e_t=0.01 and *e* represents the value of max[$|I_i^{(ls+1)}-I_i^{(ls)}|$ / $|I_i|$], which means that if $e \le e_t$, it's convergent; if $e \ge e_t$, it's not. It is found that when *N* equals 100, 200 and 500 respectively, the errors are almost the same. Figure 2 shows that when the iterative step is larger than 4, *e* is smaller than 0.01. Therefore, the convergence condition of Gauss-Seidel iteration scheme is well satisfied.

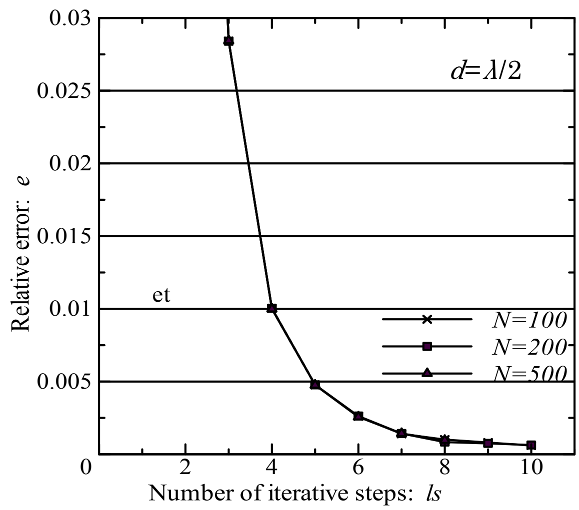

Fig.2 The relative error versus the number of iterative steps.

Figure 3 shows that relation between relative error *e* and array spacing *d*. It indicates that when *d* is much smaller than 0.5 λ , the relative error is much larger than e_t and obtained current does not converge. When array spacing is close to 0.5 λ , the relative error *e* becomes very small. And when array spacing *d* is larger than 0.5 but smaller than 0.8 λ , the relative error *e* becomes large. And when it's near 0.85 λ , the value goes back to be small. As a result, it can be said that the Gauss-Seidel method shows good convergence when array spacing *d* is close to 0.5 λ or larger than 0.85 λ .

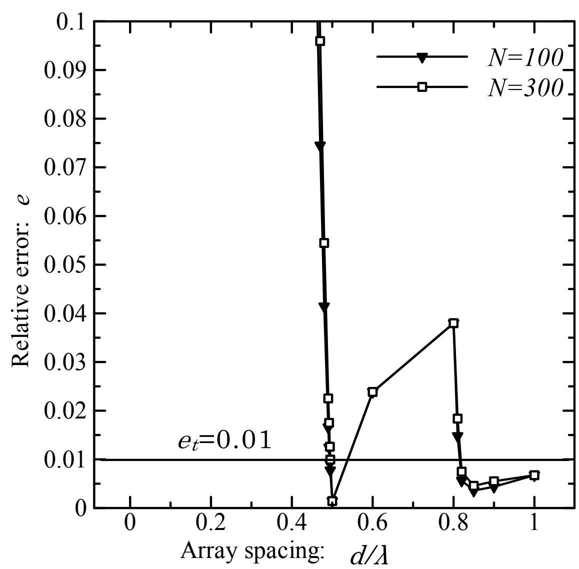

Fig.3 The relative error *e* versus array spacing *d*

4. Comparison of CPU time

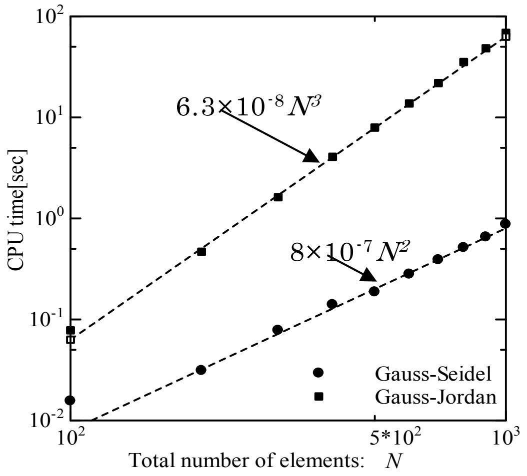

Fig.4 CPU time versus total number of elements

In order to show the validity of this method, the CPU time for solving the matrix equation versus total number of elements N is shown in Fig.12. The curve of the Gauss-Jordan method, one of the traditional iteration methods, is plotted for comparison. As expected, the CPU time is proportional to N^3 by using the Gauss-Jordan method, while it is proportional to N^2 by using Gauss-Seidel method. The cost saving effect of the numerical computation is significant.

5. Conclusion

Gauss-Seidel Scheme has been employed to solve the matrix equation of the MoM analysis for the large-scale array antenna with non-uniform sized elements. It is found that the method is simple and effective to solve this kind of model.

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